VYSOKÉ UČENÍ TECHNICKÉ V BRNĚ
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DISSERTATION THESIS

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ANALÝZA NÁVRHOVÝCH PARAMETRŮ PŘEDPJATÝCH BETONOVÝCH KONSTRUKCÍ POMOCÍ OPTIMALIZAČNÍCH ALGORITMŮ

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## ABSTRACT

During recent years more and more emphasis has been put on saving and ecological aspects of the civil engineering industry. As the total volume of concrete being produced on our planet is immense (ca $10^{10}$ tons per year), the possibility of decreasing it by even a small percentage can bring large savings in material costs, transport and other costs and reduction of $CO_2$ production and other pollution. Therefore, optimal analysis of design variables of concrete structures appears to be of high importance.

Optimization is finding the best solution to a given problem. Many disciplines define different optimization problems and it is typically the minimum or maximum value of the objective function that is searched. It is known that mathematical procedures and algorithms to find an optimal structural design are used in practice in mechanical engineering, but the use of these tools in civil engineering is rather exceptional. Generally, scientific works deal with the optimal design of structures only. Finding of an optimal shape and dimensions is usually a question of the engineer's experience and good "guess", which is then verified by calculation. There are many reasons explaining why optimization in common practise is used only occasionally. One of them is the absence of proper user friendly software tools which could help within relatively short time available for structural design. Another reason is the complexity of optimization tasks as well as a lot of constraints in civil engineering design codes. Last but not least, the change of design variables of buildings, bridges and structures of special types do not express regular response. This issue is discussed in the submitted work.



## KEYWORDS

Concrete structures, prestressing, design variable, optimization, genetic algorithm, costs, post-tensioned bridge, cable-stayed structure.

**ABSTRAKT**

V posledních letech je stále více kladen důraz na úspory a ekologická hlediska ve stavebnictví. Celkové množství vyprodukovaného betonu na zemi je obrovské (odhadem $10^{10}$ tun za rok). Možnost snížení jeho výroby pouze o několik procent může přinést významné materiálové úspory, redukci výdajů na přepravu a s tím souvisejících nákladů. Mezi další aspekty je možné zařadit omezení produkce $CO_2$ a jiných škodlivin. Z těchto důvodů se jeví optimalizační analýza návrhových parametrů betonových konstrukcí jako velmi důležitá.

Obecně lze definovat optimalizaci jako nalezení nejlepšího řešení pro danou úlohu. Optimalizační úlohy jsou definovány pro různá odvětví a spojuje je hledání minima nebo maxima cílové funkce. Je známo, že matematické metody a algoritmy umožňující nalezení optimálního tvaru nebo parametrů konstrukce, jsou již řadu let používány ve strojírenství. Každodenní aplikace ve stavebnictví je však doslova výjimečná. Návrhem stavební konstrukce pomocí optimalizačních algoritmů se zabývají pouze práce na vědecké úrovni. Nalezení optimálního tvaru konstrukce je obvykle otázkou zkušeností a znalostí projektanta, který návrh následně ověří svým výpočtem. Existuje mnoho důvodů, proč nejsou tyto algoritmy používány v běžné praxi. Patří mezi ně zejména absence uživatelsky přístupného a srozumitelného programu, který by napomáhal zoptimalizovat konstrukci v relativně krátkém čase, a také složitost optimalizační úlohy. Dalším limitujícím faktorem je skutečnost, kdy stavební konstrukce jsou vystaveny mnoha omezujícím podmínkám požadovaných normou. A v neposlední řadě pak změna návrhových parametrů budov, mostů či speciálních typů konstrukcí nevykazuje pravidelnou odezvu. Výše uvedená problematika je náplní předložené disertační práce.

**KLÍČOVÁ SLOVA**

Betonové konstrukce, předpětí, návrhové parametry, optimalizace, genetické algoritmy, náklady, dodatečně předpjatá mostní konstrukce, zavěšená konstrukce.

## REFERENCE

DLOUHÝ, Lukáš. *Analysis of design variables of prestressed concrete structures using optimization algorithms*, Brno 2012, 165 p. 171 with app., Dissertation thesis, Brno University of technology, Faculty of Civil Engineering, Institute of Concrete and Masonry Structures. Supervisor doc. Ing. Jaroslav Navrátil CSc.

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DLOUHÝ, Lukáš. *Analýza návrhových parametrů předpjatých betonových konstrukcí pomocí optimalizačních algoritmů*, Brno 2012, 165 s. 171 s. s příl., Disertační práce, Vysoké učení technické v Brně, Fakulta stavební, Ústav betonových a zděných konstrukcí. Vedoucí práce doc. Ing. Jaroslav Navrátil CSc.

**Statement:**

I declare I worked out this dissertation thesis by myself using technical literature mentioned in the bibliography.

Brno, 8. August 2012 Ing. Lukáš Dlouhý

**Prohlášení:**

Prohlašuji, že jsem disertační práci zpracoval samostatně a že jsem uvedl všechny použité informační zdroje.

V Brně dne 8. 8. 2012 Ing. Lukáš Dlouhý

## ACKNOWLEDGMENTS

This dissertation thesis has been written at the Institute of Concrete and Masonry Structures which is a department of the Faculty of Civil Engineering in Brno, University of Technology. I would like to thank the Faculty for the opportunity to write and finalize the thesis.

I would like to express my deepest personal gratitude to my supervisor doc. Ing. Jaroslav Navrátil CSc. for his invaluable help, support and beneficial contribution during the process of thesis writing.

Furthermore, I wish to convey my appreciation to the company Nemetschek Scia for providing a licence of efficient software for the analysis, design and check of the structures. I would particularly like to thank my former colleague Ing. Jaromír Kabeláč for an excellent contribution during the implementation of the mathematical algorithms needed for the optimization.

Next, my thanks go to Ing. Matěj Lepš, PhD. from Czech Technical University in Prague for collecting and providing all necessary documents related to previous works about optimization of prestressed concrete structures.

Finally, this thesis has been written with the financial support of the state budget provided by the Ministry of Industry and Trade of the Czech Republic, specifically within the project FT-TA4/100 – “Optimization as a tool of the sustainable development and increasing competitiveness of the Czech civil engineering”.

## CONTENTS

# 1 INTRODUCTION

Prestressed concrete, as a structural material first described by E. Freyssinet, has been used on a daily basis more than 80 years. Through that time uncounted numbers of the prestressed bridges, building slabs and silos have been built. The basic principle of loading transmission in reinforced concrete is that reinforcement carries tensile forces and concrete resists to compressive stresses. Tensile forces can be caused by external load, temperature load or time dependent effects like creep and shrinkage. Although the compressive strength of concrete is large, the corresponding tensile strength is minimal. Therefore, the tensile forces should be transmitted by reinforcement – passive (nonprestressed) or active (prestressed). The prestressing reinforcement is tensioned against the concrete member and introduces permanent compressive stress in the structure. The application of prestressing (p) causes the creation of compressive reserve in the structure, see Fig. 1.1. This behaviour minimizes the effect of permanent (g) as well as variable (q) loads.

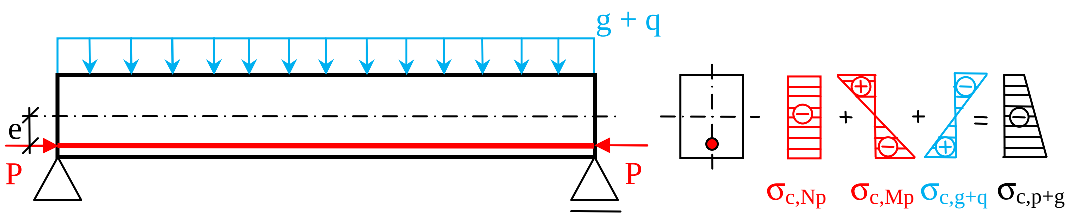



Fig. 1.1 Stress distribution in a prestressed section [7]

An active modification of distribution of internal forces caused by prestressing is another advantage of prestressed concrete. The effect is clearer in the case of transforming the prestressing load by the system of forces acting along the beam. An engineer is able to influence distribution along the beam very easily by changing the tendon geometry, which is significant mainly in a statically indeterminate structure. Nowadays high strength materials are widely used in civil engineering. In fact, the prestressing strand is also of high strength steel. The prestressing of concrete structures with high strength materials leads to the decrease of the cross-section depths. This is important mainly in case of long-span bridges, where the selfweight of the structure is the dominant load. The reduction of the cross-section is suitable from the functional, aesthetical and economical points of view.

Two basic types of prestressed concrete can be distinguished, (1) pre-tensioned and (2) post-tensioned. The difference between these types lies in the application of prestressing force. Pre-tensioned members are made in factories, where prestressing reinforcement is stressed and temporarily fixed in anchorage blocks. Next, concrete is cast into stressing beds. Afterwards it is hardened and prestressing is applied automatically to the element by releasing the bond between concrete and reinforced strands. The bond effect is the main idea of the pre-tensioned concrete. On the contrary a post-tensioned element is first cast. Only after hardening the prestressing is applied to the member and permanently anchored by the anchorage devices.

## 1.1 Design approaches

When we speak about analysis of structures, two design approaches can be distinguished. The deterministic one is based on exact values of all parameters. Oppositely, the stochastic approach is based on the probability of the input values and the calculation of overall reliability of the structure [9]. The probability of failure and corresponding reliability index are theoretical values that are used for calibration of the codes, but they should not represent the existing level of failure. The using of fully probabilistic methods requires knowledge of the statistic and probabilistic methods and procedures. The statistic distributions of the material characteristics, load or dimensions are needed for using of probabilistic methods. These values are not usually accessible and thus stochastic methods are applied widely in the design routine.

Several design methods has been described in the past. As the knowledge about concrete structures has been developed also the design approaches has improved. The design codes and methods became writing at the beginning of 20th century. Beforehand structures were designed based on experiences, knowledge and intuitive of engineer who had very illustrative information about load, materials and structure behaviour.

The method of allowable stresses was very popular to half of the last century. We can mention Czech national code for design of reinforced bridge structure ČSN 73 6206 [131] or Allowable Stress Design in ACI318 [129]. Ratio of maximal calculated stress ($\sigma_{max}$) to allowable stress ($\sigma_{lim}$) has to satisfy deterministic based limit.

$$\sigma_{\max} \leq \sigma_{\lim}. \tag{1.1}$$

Next, safety factor design method was based on principle calculated safety factor ($s$) is higher than limit ($s_0$). Procedure seems to be more objective due to covering overall values of resistance ($X_{cap}$) and load effects ($X_{load}$). Impossible including of reliability effects (as in allowable stress method) is the biggest disadvantage too. One of the codes using safety factor was Czech design code for bridges (ČSN 73 6207 [132]) valid until 03/2010.

$$s \leq s_0; s = X_{\mathrm{cap}}/X_{\mathrm{load}}. \tag{1.2}$$

Nevertheless, the most of serious accidents causing structural collapse are due to non-stochastic reasons, especially human errors. An unpredictable danger should be covered by the safety boundaries. This can be ensured by introducing of the overall or by partial safety factors decreasing the danger to allowable limits. New conception of design was prepared based on probability approach of limit state. Limit state theory was successor of mentioned methods. Structure needs to satisfy two limit states (1) ultimate limit state and (2) serviceability limit state. Practical application of this method requires using of partial safety factors which compares design values of maximal load ($E_{\mathrm{d}}$) to design (minimal) resistance of structure ($R_{\mathrm{d}}$). The most of current valid design codes used this method (LRFD [128], Eurocodes [133]).

$$E_{\mathrm{d}}(F_{\mathrm{d}}; f_{\mathrm{d}}; a_{\mathrm{d}}) > R_{\mathrm{d}}(F_{\mathrm{d}}; f_{\mathrm{d}}; a_{\mathrm{d}}). \tag{1.3}$$

However, it is probabilistic method, the results do not give imagine about reliability reserve or reliability factor. Partial load factors were estimated based on statistical distributions, theory of probability and the tests but the many simplification of problem were accepted. Nowadays lot of publications dealing with optimization are related to ACI and Eurocode which both are based on limit state with partial safety factors.

Generally, the optimal design of prestressed concrete can be divided into two basic groups. The first is Load-balancing method presented by Lin [74]. The balancing of certain part of permanent or (and) variable can be transferred by the effect of prestressing. The second group is Magnel theory of the allowable stresses. Magnel [81] constructed the diagrams of possible configuration of the level of the reciprocal values of prestressing forces and eccentricity of tendons. Such kind of graphs gives to engineer overview of usability and availability of the designed parameters of the structure.

## 1.2 Design processes

Generally, a persisting economic crisis and increasing costs of concrete and steel material can also play an important role in the design of the structure as required from an investor. A designer is able to perform several manual calculations with different parameters (cross-section dimension, number of strand etc.), nevertheless, the optimal configuration of parameters will not ever be obtained without using a sophisticated optimization tool. When designers speak about optimal solutions, it is usually the optimal design of some particular parts of the structure. This leads to the optimization of cross-section dimensions, minimization of the required area of reinforcement or the optimal tendon geometry along the beam. A particular member or detail is optimally designed, but the global optimization design may not be satisfied from different points of views such as minimal cost, construction time or labour.

Two basic types of design processes can be distinguished, (1) traditional and (2) automatic. The first traditional process is based on the designer's calculation of cross-section dimensions, necessary areas and other values according to explicit mathematical formulas. In the initial design this is not sufficient due to some reason and so they have to repeat, the whole design process. They are guided by their experience, knowledge and intuition. Usually this process requires tens of manual calculations with uncertain results. Time and labour costs are significant and the optimal design depends only on the designer's skills. A typical scheme of the traditional designer process is explained in Fig. 1.2.

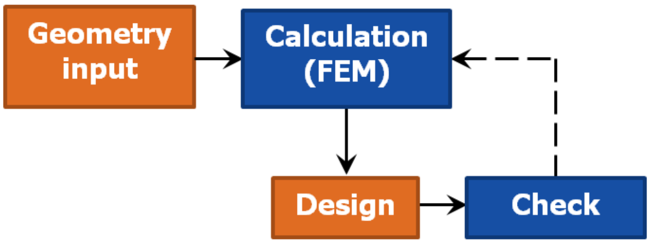



Fig. 1.2 Traditional design process, FEM (finite element method)

The automatic design process is an alternative to the traditional one. Generally, the design "trial and error" procedure is replaced by an automatic calculation controlled by the sophisticated methods based on very efficient mathematical algorithms. These algorithms are able to evaluate a set of parameters providing such configuration of the structure that implies minimal structural costs, constructions and serviceability requirements and others. An optimal design of the structure is ensured from the standpoint of selected costs together with functional and resistance expectations. The use of this process is suitable for several reasons. The main one consists in the very high capacity and performance of computer technology. The automatic process is visible in Fig. 1.3. The detailed overview of optimization of different prestressed concrete structures are mentioned in Chapter 2.4.

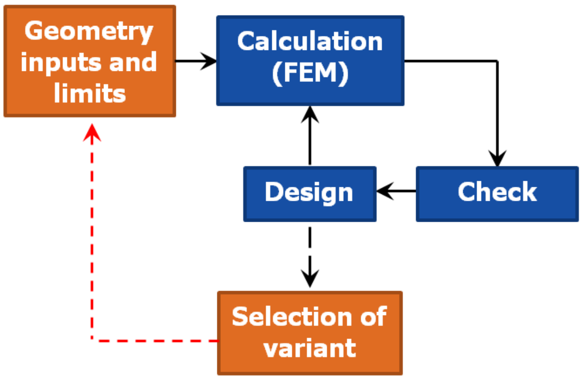



Fig. 1.3 Automatic design process

## 1.3 Organization

This thesis is divided into ten chapters and one appendix. Chapter 1 describes the main motivation of the prestressed concrete optimization, explains design approaches and processes, and comments the organisation of the work.

In Chapter 2, the basic terminology related to the optimization problems are briefly specified. The optimization is categorized into several groups according to the type of the optimization problem. Furthermore, this Chapter summarizes previous work on the optimization of prestressed concrete structures. The first part distinguishes optimization according to the objective function. Afterwards, the optimizations of different types of prestressed concrete structures like prestressed beams, columns and slabs are mentioned. The last parts of this Chapter recapitulate optimization of prestressed bridges and special prestressed structures.

Chapter 3 defines the main goals of this thesis. Next, Chapter 4 explains used optimization algorithms from the mathematical point of view. An optimization procedure is described for genetic algorithm, heuristic and gradient based methods. Then, Chapter 5 illustrates a process flow of developed optimization tool.

The application of mathematical methods is validated on a specimen in Chapter 6. This has been done on a simply supported post-tensioned beam. Basically, two types of geometries are analyzed. At the end of the chapter, a sensitivity analysis of different inputs is performed.

Then, Chapter 7 concerns the use of optimization method in the design of tendon geometry of post-tensioned concrete bridges. Optimization makes it possible to reduce the required number of the tendons in typical post-tensioned concrete bridges. The deck, double T section and three girder bridges are optimized. The information provided in

Chapter 8 describes the optimization of the tendons in the floor of a shopping centre. Prestressing is used to eliminate of the internal columns used in the floor. The application of optimization algorithm is illustrated for a cable-stayed structure in Chapter 9, which contains comments on basic requirements for the design and analysis. A special part describes different types of analysis with their advantages. The usability of the optimization algorithm is verified on a simple cable-stayed structure for particular calculation type.

The conclusions of this work together with recommendations for further research are presented in the last Chapter 10. An appendix provides information about necessary settings for particular optimization algorithm.

# 2 THE STATE OF THE ART

Nowadays, the prestressed concrete is used in wide range of production. Different types of beams, columns, slabs and special structures are produced using prestressed concrete. The same division is used for the survey in Chapter 2.6. Next, prestressed concrete can be divided into two groups (1) pre-tensioned concrete and (2) post-tensioned concrete. The main application areas of pre-tensioning are prefabricated beams. The importance is significant in the prefabricated producers because very small saving of material in one element causes a huge decrease of costs in overall. Post-tensioning can occur in any type of a structure. Principally post-tensioned concrete bridges and building floors are used. Therefore only these two types of prestressing of beams are correspondingly shown in sections 2.6.1.1 and 2.6.1.2.

The optimization of the prestressed concrete structures can be seen from different views: the shape of the structure, the process of prestressing, the type of an analysis, an objective function, selection of the variables and the optimization procedure, etc., which are explained in Chapter 2.1. The main important parameter is the shape and therefore, this will be the main division point.

The construction of the prestressed concrete structures is affected by many aspects such as concrete properties, reinforcement parameters and formwork. By their change, the optimization process can be formulated, e.g. in the terms of the weight of concrete and steel, area of tendons and many others. To get more reasonable results, the total cost of the structure should be optimized as will be shown in details.

Before the review literature, different objective function and optimization of particular structural types is commented, the basic terms are explained in the following chapters.

## 2.1 Basic terms

Generally, the majority of practical examples have more than one solution. Optimization means to find the best of them with respect to some effective criteria. The search of an extreme of the function with more variables is characteristic for the optimization process. In this chapter, the main objective is to explain several fundamental terms related to the optimization task. These are the objective function, constraint, extreme, robustness and others.

### 2.1.1 Definition of the optimization task

Generally, optimization is defined as searching the minimum (maximum) of the investigated function. From the mathematical point of view, the optimization problem is to find a vector of design variables showing a minimum (maximum) with respect to complying constraints. This explication can be expressed as follows

$$\begin{aligned} & f(\mathbf{x}) \;=\; \min(\max)\,, \\ \text{s.t.}\quad & g_{\mathrm{j}}(\mathbf{x}) \leq 0 \quad j = 1, 2\ldots m\,, \\ & l_{\mathrm{j}}(\mathbf{x}) \leq 0 \quad j = 1, 2\ldots p\,, \end{aligned} \tag{2.1}$$

### 2.1.2 The objective function

The target of the optimization process is the best configuration of certain parameters (design variables) satisfying constraints. Generally, it is possible to find more solutions fulfilling the initial requirements. Nevertheless, the goal is to find the best one. The criterion for evaluating the set of parameters must be established at the beginning of the optimization. This criterion is called objective function $f(\mathbf{x})$. The objective function is usually selected as a minimal area of cross-section, the minimal amount of reinforcement or minimal total costs. Where there is one objective function for the investigated problem this task represents single-objective optimization. Multi-objective optimization enables to optimize more objective functions in one step (for instance minimal costs and maximal allowable deflection). The formula for multi-objective optimization is expressed as follows:

$$f(\mathbf{x}) = \alpha_1 \cdot f_1(\mathbf{x}) + \alpha_1 \cdot f_2(\mathbf{x}). \tag{2.2}$$

Where $\alpha_1$; $\alpha_2$ are weighted constants for each objective function $f_1(\mathbf{x})$ and $f_2(\mathbf{x})$. The result is a set of the Pareto–optimal solutions [96].

### 2.1.3 Constraints

In general, in addition to the objective function, it is usually necessary to define additional functions providing a reasonable solution for the optimization problem. These functions are known as constraints. Constraints can be of different types (geometrical, checks and special requirements).

The function $g(\mathbf{x}) \leq 0$ expresses constraints and value $g(\mathbf{x}) = 0$ distinguishes the solutions to permissible ($g(\mathbf{x}) \leq 0$) and non-permissible ($g(\mathbf{x}) > 0$). There are two basic types of the optimization process varying from each other by the existence of constraints. In the first type, there is at least one constraint and the second is unconstrained. Constraints are usually expressed in the normalized form, see the following formula for limitation of compression stress in concrete

$$g(\mathbf{x}) = \frac{\sigma_{\mathrm{cc}}}{\sigma_{\mathrm{cc,lim}}} - 1 \leq 0 \ . \tag{2.3}$$

Special penalization functions have been developed for working with constraints. These functions transform the problem of constrained optimization by sets of unconstrained optimization. The evaluating fitness function is decreased according to the intensity of constraint exceeding. Therefore, these functions penalize an excess of constraints. The selection of functions has to respect several conditions such as annual values for accepted solution and positive values for refused solutions. The dependence of the penalization function on the constraint function is shown in Fig. 2.1. Different shapes of the objective function are commented in Chapter 2.2.

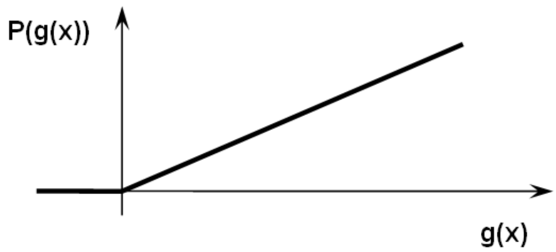



Fig. 2.1 Penalization function

### 2.1.4 Global and local extreme

The target of optimization is to find the best possible solution to a problem. From the mathematical point of view, it is the search of a global extreme of the objective function. In parallel to the global extreme, there are also local extremes. The convergence of the method to them usually has a negative effect. Several optimization methods have the difficulty that they neglect the search of a local extreme as if it were a global extreme. A gradient based method (see Chapter 4.2 or [50]) can fail in this context. The difference between the local (E1) and global (E2) extreme of function *f(x)* is shown in Fig. 2.2

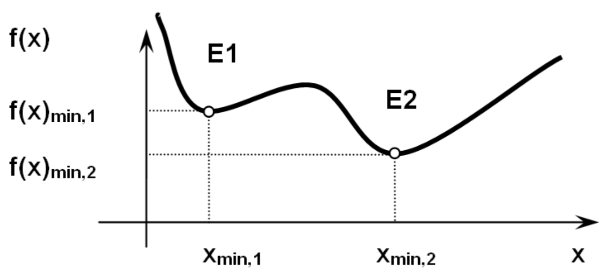



Fig. 2.2 Comparison of the local and global extreme

### 2.1.5 Robustness

The distinction between the local and global extremes is an important characteristic of the method known as *robustness*. This is defined as the capacity to find the global extreme of the objective function. More robust methods find the global optimum with a high degree of probability.

From the standpoint of robustness, two optimization groups exist

- gradient based method and
- objective function values method.

The selection of the initial value of parameter $x_0$ for optimization is also a very important step. A gradient based method from point P1 ends in local minimum (see Fig. 2.3).

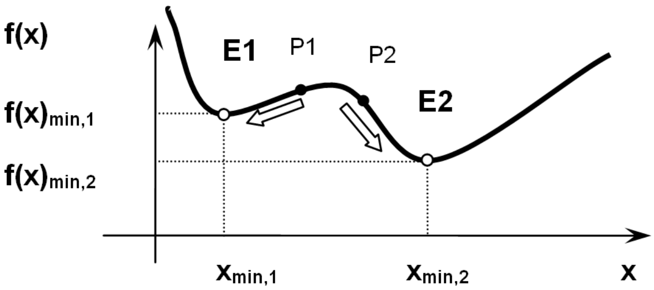
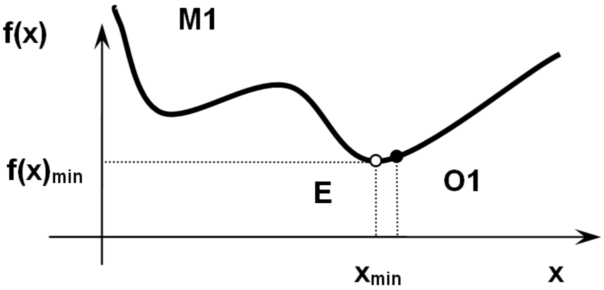



Fig. 2.3 Robustness of method

### 2.1.6 Accuracy

Another important characteristic of the optimization method is its accuracy. The application of different optimization algorithm for one problem gives usually leads to different optimal solutions. Accuracy can be defined as the ability to find a solution close to the global optimum. For instance, genetic algorithms (see Chapter 4.1 or [114], [115] and [82]) are described with high robustness but with low accuracy. Gradient based methods show the opposite behaviour. In the following figure (see Fig. 2.4), it is possible to see optimum finding O1 using method M1, which is more accurate than optimum O2 found by method M2.



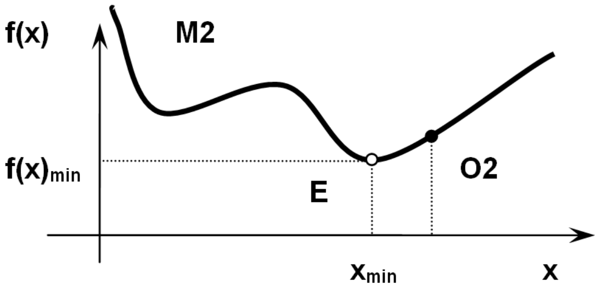



Fig. 2.4 Accuracy of the optimization method

The type of the optimization method can be selected according to the type of the optimization problem. It is possible to determine the selection on the following basis:

- When we perform an improvement of a known good solution gradient based methods are suggested.
- When we do not know any solution beforehand then more robust method like genetic algorithm should be used.

The combination of the robust and accurate method offers a very efficient tool. The robust method enables to find a solution near the global optimum which is the starting point for the accurate gradient method.

### 2.1.7 Parameterization

Optimized design variables are for instance the dimensions of the cross-section, the number and geometry of the tendon and optimal position of supports, etc. Previously these items have to be parameterized. Parameterization is one of the most important steps during the whole optimization process. The obtained solution depends on the limits of parameter dependency. Two basic cases can appear.

- Insufficient parameterization – optimization algorithm cannot find the optimal configuration of the structure.
- Redundant parameterization – large dependency between many parameters leads to a very slow optimization process and huge amount of iterations.

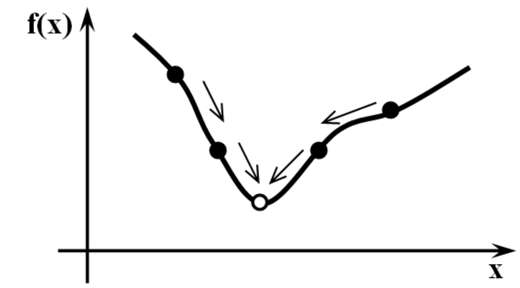
### 2.1.8 Definition of the optimization task

Generally, the correct formulation of the optimization problem is an important step of the optimization process. Optimization examples usually require a detailed model of the structure and a suitable selection of design variables. Dependently of the type of optimization problem and its complexity, it is possible to define tens of design variables and more objective functions. Constraints usually affect geometrical dimensions of the structure, stresses, material failure, etc. Therefore it is necessary to evaluate suitable algorithm for each optimization task, select correct objective function and set effectively the constraints. Consequently, algorithms can be summarized according to several characteristics:

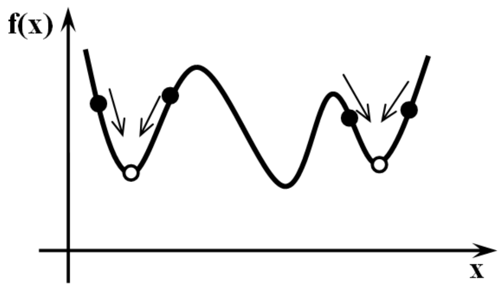
- Reliability and robustness – the algorithm ensures convergence to optimum,
- Generality – the algorithm solves different types of problem independently of constraints or the type of objective function,
- Effectiveness – the algorithm searches optimal solution within minimal number of iterations; nowadays parallelization of computers can be used,
- Simplicity – the algorithm is understood both by an optimization expert as well by a standard engineer. This is a very important factor regarding daily use in practice. An algorithm requiring a lot of engineer input in each optimization task is almost useless in daily work.

## 2.2 Objective function shape effect

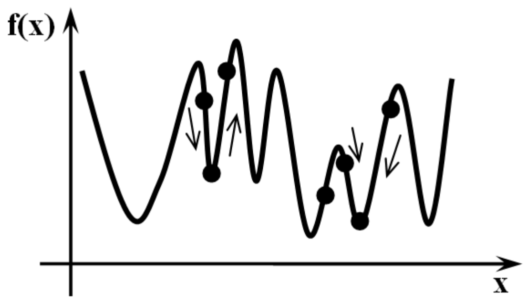
The shape of the objective function is an important factor for achieving an optimal solution. Generally, there are many different forms of the objective function (see [123]). Dependently of this, a particular optimization algorithm has to be used. The simplest example occurs when only one extreme exists; see Fig. 2.5 (a). In the case of multiple local extremes, to find the global optimum with some methods is difficult. In particular, gradient based methods are not efficient. The initial configuration of parameters is decisive for finding local extremes; see Fig. 2.5 (b). When the objective function consists of more local extremes, gradients methods cannot be used; see Fig. 2.5 (c). Sometimes, the gradient method can provide misleading information about the global extreme when the process focuses on the prevailing part going to the local extreme, see Fig. 2.5 (d). The most problematic cases relate to very shallow objective function (see Fig. 2.5 (e)) and function including extreme in several points only; see Fig. 2.5 (f). The previous two figures usually represent objective functions of prestressed concrete structures within their constraints. Finally, it s very complicated to find the global extreme of the task in the last two examples. The gradient based method cannot be used at all. Nevertheless, the application of the evolutionary algorithm is very promising.



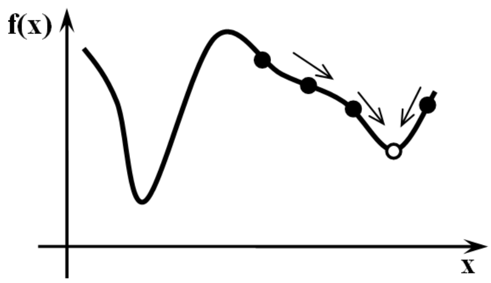

(a) Global optimum



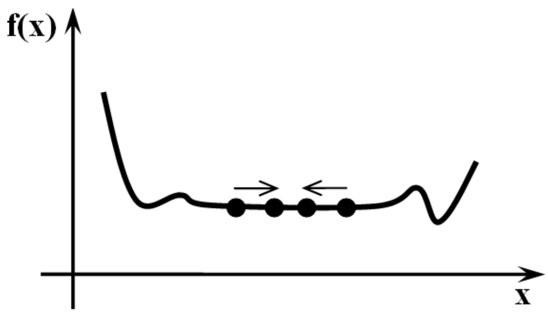

(b) Multiple local optimum



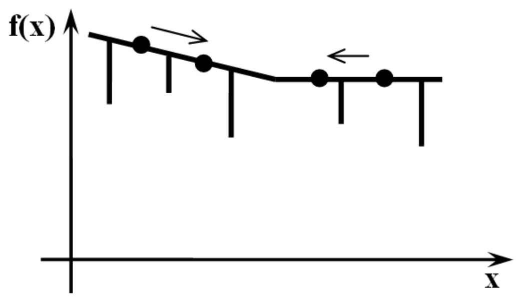

(c) Rugged optimum




d) Deceptive optimum




(e) Neutral optimum




(f) Nightmare

Fig. 2.5 A possible shape of the objective function (minimization) [123]

## 2.3 Categorization of optimization

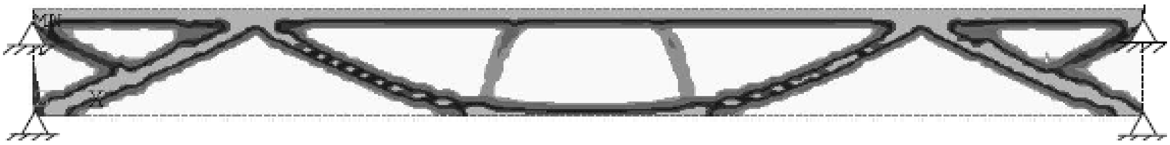
Generally, the optimization of structures can be divided into categories according to the obtained result. Steven [113] distinguishes the following groups:

- topology optimization,
- shape optimization,
- size optimization,
- layout optimization and
- topography optimization.

### 2.3.1 Topology optimization

The first optimization category is applied when the overall shape of the structure is unknown. Only optimization criteria, limits for design variables and constraints are given in advance. It is frequently used in the aircraft and automobile industries. A typical case is the design of reinforcement in concrete cross-sections in civil engineering. The cross-section dimensions are predefined and the reinforcement amount within its position is optimized. Another example can be the design of a truss girder where distribution of the members and their position is searched. The last example represents the volume of material supported in certain points. The topology optimization enables to find the optimal distribution of material, see Fig. 2.6.

a) initial volume of material



b) optimal material distribution

c) Villa Bedretto Bridge, Switzerland

Fig. 2.6 Example of topology optimization [112]

Generally optimization focuses on minimization of the total mass of material or total costs. Methods belonging to this category are Optimality criteria, Homogenization, Evolution structural optimization, genetic algorithm or cellular automata.

### 2.3.2 Shape optimization

The shape optimization is efficient where the overall topology of structure is known, nevertheless, it needs an improvement in geometry. Two basic cases of shape optimization are distinguished (a) global and (b) local. Global shape optimization deals with optimum shape of the whole structure. An enhancement of some structural details can be observed in local shape optimization. The shape optimization searches the optimal shape of structure depending on the redistribution of stresses. A typical example can be the optimization of node positions in a steel truss girder (see Fig 2.7). Usable methods for this kind are gradient based, simulated annealing and genetic algorithm.

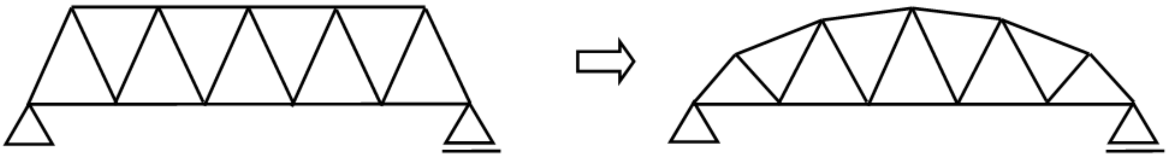


Fig. 2.7 Example of shape optimization

### 2.3.3 Size optimization

Basically, a structure is defined with coordinates of nodes, dimensions and cross-sections for size optimization. The combination of the above mentioned parameters enables to achieve optimal criteria. Predominantly it deals with the optimization of the cross-section. Standard Autodesign is a typical case of size optimization. Generally, Autodesign is very fast heuristic algorithms efficient for a statically determinate structure. Unfortunately, it fails due to oscillation in statically indeterminate structures. The selection of cross-section from the library and iterations between values result to optimum. Dependently of the type of design variables, we can distinguish two cases.

- <u>Structure with discrete variables</u> – the structure consists of particular members from a predefined fabrication list (for instance rolled cross-section of steel structures, predefined reinforcement pattern in the concrete cross-section etc.). Simulated annealing or genetic algorithms are efficient methods.
- <u>Structure with continuous variables</u> – the dimension of the whole structure are independently changed (for instance weld width in steel structure). Gradient based method can be used for this type.

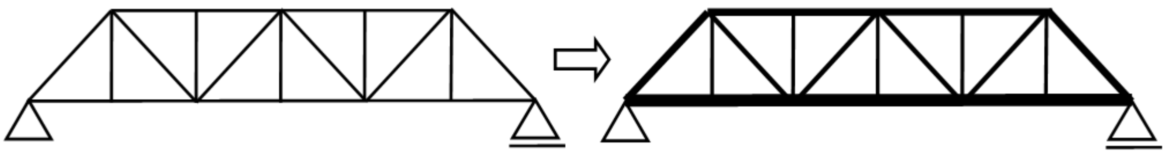


Fig. 2.8 Example of size optimization

### 2.3.4 Layout optimization

This kind of optimization can be considered as a transition between all three mentioned types before. The basic topology of structure is known in the beginning. An aim of this type is neglect ineffective unloaded parts of structure, see [63]. As an example the optimization of steel truss girder is illustrated in Fig. 2.9.

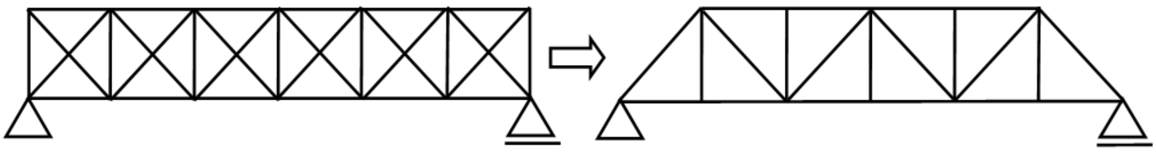


Fig. 2.9 Layout optimization of bridge structure [20]

### 2.3.5 Topography optimization

The last category is the least tested and examined on. It can be used for searching of optimal shape of membrane elements and tent steel truss structures.

## 2.4 Review literature on optimization of prestressed concrete structures

According to the study by Cohn and Dovitzer [24] on general structural optimization, there have been about 150 books and 2500 scientific articles published in years 1960 to 1994. We can roughly estimate that there are two times more now. Optimization of concrete structures composes approximately 3-5% of these numbers. A subgroup of the optimization of concrete structure is focused on prestressed concrete which is about 1/3 from all concrete optimization articles. The cause is clear. Concrete structures are very complex accompanied with many discrete variables, dozens of constraints required by codes and several types of nonlinearities.

A few articles focused on reviews of concrete structures optimizations have been published in the past. Arora [15] presented a different view of investigation and made an overview based on the type of optimization methods before 1990. Sarma and Adeli [108] reviewed optimization of reinforced and prestressed concrete according to type of structure. This categorization is the most popular in the reviews. The same authors described cost optimization of concrete and steel structures using different kind of optimization techniques in their book [1]. Girgis and Tadros [43] made overview of precast concrete bridges in US. Hassanain and Loov [47] focused on review about three levels optimization (component, layout and system) of prefabricated concrete bridges including life cycle cost optimization too. Dlouhý, Lepš and Novák focused on review related to reinforced concrete frames [33].

A review strictly focused on optimization of prestressed concrete structures has not been found. The presented chapter deals with optimization according to type of structure. The optimizations made on pre-tensioned and post-tensioned beams are described in section 2.6.1.1 and 2.6.1.2. Several articles are commented on prestressed columns (2.6.2) and slabs (2.6.3). Last two parts are focused on prestressed concrete bridges (2.6.4) and special prestressed type of structures (2.6.5).

From the numbers presented above, the rough estimate is about 50-100 articles dealing directly with optimization of prestressed concrete structures. We were able to find 80 contributions (articles, text and software) which are studied in the following sections. Unfortunately 10 searched articles were not possible to obtain.

## 2.5 Objective functions

Optimization of prestressed concrete structures can be seen from different explication of objective function. The weight of structures is the one of main criteria for optimization of concrete structures. The prestressed concrete structures are composite materials. At least three different kinds of materials are used during production (concrete, non-prestressed reinforcement, prestressed reinforcement). More objective view is to perform cost optimization which covers cost for the whole construction. Cost for formwork, transport, construction and erection can be important beside the overall weight of the structure. Interesting work is done by Naaman [90] who tried to perform comparison study between using optimization of the cost and optimization of the weight of the structures. In work of Dlouhý and Novák [32] was cross-section geometry fixed and the resultant prestressing force (area of prestressing reinforcement multiplied by an initial stress) in the element was optimized. Typical case of multiobjective optimization represents minimization of cost together with maximal initial camber performed by Lounis and Cohn [77]. To cover all aspects in the optimization problem can be done using life-cycle cost optimization. Here, the global influence of all implications as cost for maintenance, reconstruction is expressed. Ramanathan et al [101] performed study based on costs related to design and build of multiple bridge structures in Malaysia. They expressed optimal design of bridge using bridge component ratio (cost of particular part of the bridge).

### 2.5.1 Cost optimization

The cost optimization is the most studied type of optimization. Adeli and Sarma described cost optimization for concrete and steel structures in their book [1]. A review on articles focused on the cost optimization of prestressed cornet structures is performed in the following text of this paper. The most journals investigated cost optimization of prestressed beams or beam bridges. A much less are dedicated to slabs, columns and special structures. Cost optimization formula is general for all kinds of structure. Only several parts are neglected. The formula of the objective function for a material cost of a structure ($C_m$) of a general type of optimization used for prestressed concrete can be expressed as the following

$$C_{\mathrm{m}} = C_{\mathrm{c}} + C_{\mathrm{lr}} + C_{\mathrm{sr}} + C_{\mathrm{p}} + C_{\mathrm{f}}. \tag{2.4}$$

This formula is a sum of the cost for concrete ($C_{\mathrm{c}}$), cost for nonprestressed (longitudinal ($C_{\mathrm{lr}}$) and shear ($C_{\mathrm{sr}}$) reinforcement), cost for prestressing reinforcement ($C_{\mathrm{p}}$) and cost for a formwork ($C_{\mathrm{f}}$). There are more additional costs that can be added to the objective function, see e.g. Adeli and Sarma [1]. Cost for transportation, construction and erection can be also included in this initial structural cost ($C_0$).

$$C_0 = C_{\mathrm{m}} + C_{\mathrm{trans}} + C_{\mathrm{constr}} + C_{\mathrm{er}}. \tag{2.5}$$

### 2.5.2 Life-cycle cost optimization

Another example of a cost optimization is intention on life-cycle costs. The total cost includes initial cost, maintenance and inspection costs and cost for a reconstruction. Social user cost can be another group of costs which is focused on users of structures. Generally result of regular maintenance, bigger reconstruction which requires detour traffic journey, using substitute spaces, delay for passengers, and damages of cars on bridges or equipment in the building can be covered by this cost. The total cost can be quantified by individual costs in each construction or serviceability stage. A formula_for calculation can be expressed as a sum of structural ($C_{\mathrm{SC}}$) and functional ($C_{\mathrm{FC}}$) costs

$$C_{\mathrm{TOT}} = C_{\mathrm{SC}} + C_{\mathrm{FC}}. \tag{2.6}$$

A structural cost can be divided into a sum of partial costs of an initial structural cost ($C_0$), regular inspection cost ($C_{\mathrm{I}}$), maintenance cost ($C_{\mathrm{M}}$), reconstruction cost ($C_{\mathrm{R}}$) and a cost caused by the failure of the structure ($C_{\mathrm{SF}}$)

$$C_{\mathrm{SC}} = C_0 + C_{\mathrm{I}} + C_{\mathrm{M}} + C_{\mathrm{R}} + C_{\mathrm{SF}}. \tag{2.7}$$

Functional costs can be expressed as a difference between a failure cost ($C_{\mathrm{FU}}$) and benefits ($C_{\mathrm{B}}$) which brings the security.

$$C_{\mathrm{FC}} = C_{\mathrm{FU}} - C_{\mathrm{B}}. \tag{2.8}$$

The functional cost can be a cost connected with speed limits, e.g. vehicle load limits which produce delays and redirection of the traffic. The increase of living standard due to new structures can belong to benefits.

There are several studies describing this kind of optimizations with respect to prestressed structures. Chang and Shinozuka [23] studied effects of a life cycle cost on the total cost of the structure. Another detailed analysis is presented in the work by Jiang, Corotis and Ellis [52] where they control a cost during the service of a structure. The existing state of the structure was investigated and compared to the original state along with a necessary cost to get it into original state. Mohammadi, Guralnick, Yan [85] presented and optimization approach calculating current state of the bridge based on value index as a function of time to permit decisions to be made about reconstructions and proper works. Liu, Hammad and Itoh [75] studied rehabilitation optimization problem of bridge decks. Multiobjective optimization consisted of minimum cost and minimum deterioration degree was solved by genetic algorithm. This life-cycle cost can be an important part of a total cost but it will not be studied and commented in the next parts of this work.

## 2.6 Optimization of different types of prestressed concrete structures

There exist many structural types of prestressed concrete structures. We will point out beams, girder, columns, slabs, tanks and silos, many type of bridge structures or special structures like tubes or sleepers.

### 2.6.1 Prestressed beams

A lot of articles focused on the optimization of prestressed concrete structures deal with prestressed beams. As was mentioned before, there are two basic types of prestressed concrete beams (pre-tensioned and post-tensioned). More articles have been published related to pre-tensioned concrete beams because of their repeated manufactured production where huge saving can occur.

#### 2.6.1.1 Pre-tensioned beams

The application of prefabricated prestressed structural beams in different kind of structures is typical for their repetitive use. In that case just a small saving in one member can save a big amount in the total production. The decrease of the selfweight of the structure is very important also for transportation, erection or manipulation costs. The majority of articles reporting the optimization of prefabricated prestressed members are dealing with the bridge structures. This kind of optimization can be divided into three different levels:

- **Component optimization –** where only one particular property of one member is searched for. This is the most common reported type of optimization of prefabricated prestressed members. For instance, the optimization of dimensions of a cross-section, nonprestressed or prestressed reinforcement, is applied in this first level.

- **Layout optimization –** is focused on the longitudinal and transverse position of the members within the structure based e.g. on the number of spans, the type of the statical model (simply supported, continuous), etc. Much less works have been published on this level. One exception is the optimization of a prefabricated bridge girder system.
- **System optimization –** is the most complex level of the optimization which deals with optimization of the material, cross-section, prestressing layout, transverse position of girders and also with the overall concept of the bridges structure. The design of the bearing supports, geotechnical properties like foundation blocks and many others can be covered by this complex level.

##### 2.6.1.1.1 Component optimization

Kirsch [60] in 1972 presented minimal cost design of two spans prefabricated prestressed bridge girders. The objective function covered only the cost of concrete and prestressed reinforcement. Normal stresses of concrete and prestressing reinforcement and tendon geometry were constraints in his study. The linearization of the nonlinear optimization problem was aimed using partial solution of the linear subproblems.

Morris [87] in 1978 published an article dealing with optimization of prefabricated prestressed beams based on the methods of linear programming. The similar graphs as was made by Magnel were used for optimal design in this work. The area between the individual curves and lines is the zone which is used for determination of level of the prestressing force and tendon eccentricities.

Cohn a MacRae [27] in 1984 focused on optimized design of pre-tensioned and post-tensioned concrete beams. Ultimate bending moment capacity, allowable deflections, crack appearance ductility and fatigue together with detailing provisions according to AASHTO were constraints in this study. Their objective function included the cost for concrete, prestressed and nonprestressed reinforcement and also formwork cost. They concluded the using of partial prestressed concrete is much more economic than fully prestressed concrete. The performed several parametric studies which confirmed the beams with nonprestressed reinforcement only are more economic for high level of height/span ratio and low value of variable load. Opposite of that fully prestressed cross-sections are effective for low height span ratio and high values of variable loads. The limited prestressed concrete is suitable for intermediate situations.

Saouma and Murad [107] in 1984 presented their work about optimized design of prefabricated prestressed simply supported bridge I beams. The optimization problem was formulated for 6 geometric design variables, area of tensile, compressive nonprestressed reinforcement and area of prestressed reinforcement. The problem of constrained optimization was transferred to unconstrained optimization problem by the penalization function a solved using modified Newton method. They optimized girders of 6-42 m long spans and they saved 5-52% of cost. Using of partial prestressed was considered unimportant which was disproved in newer studies.

One year after (1985) Jones [53] formulated optimized design of prefabricated prestressed simply supported beam with box girder. Design variables were characteristic concrete strength, number, eccentricities and debonding length of individual strands. Constraints were checks of normal stresses in concrete in time of releasing of prestressing and in service, ultimate bending moment capacity, serviceability cracking moment capacity according to AASHTO. Objective function consisted of cost for concrete and cost for prestressing reinforcement.

Kirsch [58] extended in 1985 his work about optimization of prefabricated prestressed bridge I girders by the detailed analysis of constrains. He concluded it is better to solve subproblems separately with less number of design variables than the whole problem as one optimization task. The described optimization procedure runs in two levels. The first level is optimized design of prestressing force and position of all strands in cross-section. The next level is optimization of dimensions of cross-section based on the calculated values from the first level. Objective function covered cost for concrete and for prestressing reinforcement again.

Fereig [36] in 1994 published parametric study of prefabricated prestressed I girders using simplex method according to Canadian code. The similar graphs results were reported. The most important constraint for long span structures seems to be checks of normal stresses of concrete at service and eccentricities of strands. Normal stresses of concrete at transfer and ultimate bending moment capacity were concluded as the most critical for medium and short span bridges. In 1996, Fereig extended his work by parametric study of box girders [38] and prestressed I girders [39] according to AASHTO 1992 where investigated influence of design variables on span length and prestressing force. Usability of high strength concrete for optimization of prefabricated prestressed girders was studied by Fereig [37] in 1999. High strength concrete can decrease about 4-12% values of prestressing force.

Koskisto and Ellingwood [65] in 1997 published work investigated optimal design of prestressed hollow core cross-section. Their objective function included cost for concrete and prestressing reinforcement together with project cost (2.5% from production cost), quality during manufacturing and maintenance. Design variables were number and distribution of prestressed reinforcement, height and width of cross-section, depth of bottom and top flange and width of the web of hollow cross-section. In fact it was parametric study which resulted to graphs for different lengths (6-20 m) of the beams.

Girgis and Tadros [43] published in 2007 study related to possibilities of improvements of optimal design of prefabricated prestressed bridge I girders according to ACI code. The design of this girder can be optimized by (i) the increasing of the cross-section area of the bottom flange of the beam, (ii) using of welded wires for stirrups, (iii) the using of high strength concrete and (iv) dividing the girder to several parts due to maximal transportation length.

In 2010 Hernandez, Diaz and Marcos [49] presented an article dealing with optimization of prefabricated prestressed I girders. Value of prestressing force was selected as objective function. Normal stresses in concrete, curvature of prestressed tendon and maximal possible eccentricity were constraint in this example. It was possible to design different level of prestressing force and its geometry for three different cases during optimization.

Algedra, Arafa and Ismail [13] published a scientific paper in 2011 dealing with optimized design of nonprestressed and prestressed beams with genetic algorithm according to ACI318-05. Defined objective function was cost for concrete and reinforcement. Design variables were dimensions of cross-sections, numbers and diameter of longitudinal nonprestressed and prestressing reinforcement with theirs eccentricities. They saved 18% of costs using method of genetic algorithm

#### 2.6.1.1.2 Layout optimization

Torres, Brotchie and Cornell [119] in 1966 formulated the first algorithm for optimal design of prefabricated prestressed concrete bridge girder with cast-in-place deck. The objective function was based on the cost of concrete, prestressed reinforcement in girder and nonprestressed reinforcement in the deck. They covered also additional cost as transfer, manipulation or final erection costs. Design variables were depth of cross-section, number and transverse distance among girders. The optimization was performed based on the constraints of the normal stresses in concrete and deflection of the girders with deck. The graphs with dependency of span length and total cost for two different types of the load systems are results of this work.

Naaman [89] in 1972 programmed the automated procedure of optimize design of prefabricated prestressed I beams. Program performed design and selection of the optimal geometry of cross-section and prestressing layout of prefabricated I beams according to American code AASHTO and Canadian code CSA-S.6. The graphs based on the span length and transverse distance of the beams with relation to prestressing force were results of this parametric study.

Fereig [40] in 1985 linearized problem of optimal design of prestressed concrete beam for finding cross-section dimensions and minimal prestressing force according to Canadian code. Graphs for determination of minimal required area of prestressing reinforcement dependent on span length and transverse spacing of girders were results of this work.

Cohn and Lounis [25] applied the optimization in three levels on multiobjectve design of partially and limited prestressed bridge girders. The objective function was formulated on minimal cost of concrete, cost of prestressing reinforcement within design variables as maximal girder spacing, minimal height of the deck, maximal possible span length and ratio span/height of cross-section. The study above was extended [79] by the same authors for multiobjective optimization of concrete girders.

Hassanain [4], [48] published in 1998 work related to optimal design of pre-tensioned bridge girder. The results of this work are graphs for using of different typical cross-sections, numbers of girders, and dependency of prestressed reinforcement layout on girder length and width of bridge according to OHBDC. The optimization was made using parametric study of all possible solution. Objective function covered cost for manufacture, transportation and final erection to bearing blocks together with cost for monolithic slab. He concluded the using of four girders is more suitable for bridges from concrete with $f_{ck}$ = 40-80 MPa. The high strength concrete (> 100 MPa) is more economic for structure with span more than 52 m and using of three girder only. Just only 2 girders are suitable for length less than 42 m and strength between 40-100 MPa.

Sirca a Adeli [110] presented in 2005 an article which deals with optimization of total cost of pre-tensioned bridge girders structure. Their objective function consisted of cost for girders (concrete, nonprestressed and prestressed reinforcement) and cost for monolithic deck (concrete and nonprestressed reinforcement) and also cost of formwork. The example was formulated as nonlinear problem and it was solved using neural dynamic method and penalization function according to Adeli and Park. Constraints were requirements for bending moment capacity, shear capacity and minimal area of reinforcement based on the crack width, minimal depth of the monolithic deck and reinforcement percentage of longitudinal reinforcement for monolithic deck according to AASHTO 1999. This method was applied on the 36 metres long simply supported structure from prefabricated prestressed I girder.

Aydin and Ayvaz [16] presented in 2010 an article focused on topological and shape optimization of prefabricated prestressed I bridge girders using genetic algorithms. The objective function was total cost for I girders and consists of cost for concrete and prestressed reinforcement. Design variables were cross-section dimensions, area of prestressing reinforcement and number of girders in bridge. Normal stresses in concrete, deflections and geometrical limits according to AASHTO were selected as constraints. Topological optimization was used for finding optimal shape of cross-section together with optimal prestressed reinforcement layout, transverse distance of the girder and theirs number. The genetic algorithms were used for optimization and using this method they save about 28% of cost on 32 m long span and 10 m width structures.

#### 2.6.1.1.3 System optimization

Lounis a Cohn [77] presented in 1993 an article dealing about optimized design of prefabricated prestressed bridge I girders with monolithic deck. Design variables were checks of ultimate and serviceability limit states according to OHBC (Ontario 1983). Method of Lagrange coefficients was used for optimization task. The optimization consisted of three steps. The first step was optimization of the cross-section dimension, deck height and required area of nonprestressed and prestressed reinforcement together with their eccentricities. The second step was focused on longitudinal (number and length of spans) a transverse (number of girders and their spacing). They compared different structural systems as composite or hollow slabs, and different cross-sections in the third step. They used special method called sieve-search for solving of the second and third steps. The objective function was written in terms of cost for concrete, cost for nonprestressed and prestressing reinforcement and also cost for monolithic transverse end beams between prefabricated girders. Cost for manufacturing, transportation and final placing on the bearing areas were also included in the objective function. The important fact of this work is the more suitable is optimization of the whole system of structure than optimization of each part individually. The results of this work are graphs of economic span length according to type of girder and number of spans.

Aparicio, Casa and Ramos [14] created program for design of different kind of prestressed concrete structures with wide range of cross-sections types of bridge constructions. The program was able to design structure from the structural beams through the transverse end beams, bearing zone, bearing columns to the foundations according to Spanish code EHE. Graphical output with tendon geometry layout was also output of this program. The optimization methods were not implemented in this program. This article is mentioned here because the program is very complex for design of bridge structure and it helped for optimal design certainly.

#### 2.6.1.2 Post-tensioned beams

Optimization of post-tensioned concrete structure is not often used like the pre-tensioned member optimization. This conclusion is coming from complexity of such kind of structure and a huge number of design variables which are usually discrete.

Bond [22] in 1974 performed study of optimized design of post-tensioned concrete bridge girders of one, two and three spans according to British Standard. Design variables were height of cross-section, eccentricities of tendons, area of prestressing reinforcement and concrete characteristics. Nonlinear optimization methods (Powell direct search method and McCormick penalization function) were used for searching of optimal solution.

Yua, Das Gupta and Paul [124] performed in 1986 optimized design of prestressed structure of box cross-section with post-tensioned tendons according to code BS1976. Cantilever construction method was used as construction process of this structure. Design variables were value of prestressing force with its eccentricities and depth of cross-section. Objective function covered cost for concrete, prestressing reinforcement and formwork including labour costs for formwork construction and moving as multiplication of formwork cost.

Fereig a Smith [35] in 1990 made several additional graphs for optimal design of prestressed box girders according to Canadian standard (CAN3-S6-M78). There are dependency between prestressing force and span length with transverse spacing of the girders. Constraints were normal concrete stresses, eccentricity of tendon and ultimate bending moment capacity. In fact it was parametric study for 8 types of prestressed box girders and 10 hollow section slabs.

In 1991 Marks and Trochymiak [83] presented work dealing with optimized design of tendon geometry in continuous concrete three spans bridge girders using linear programming. Objective function was focused on prestressing reinforcement only. Design constraints were considered as normal concrete stresses and tendon eccentricities with respect concrete cover according to Polish code. Cross-section dimension, materials and load were fixed. They optimized geometry of tendons with 47 design variables in 8 minutes.

In 1991 Quiroga and Arroyo [99] published very similar study where optimized geometry of prestressing tendons on fixed cross-section dimensions. Normal stresses of concrete were constraint in this case again.

Erbatur, Al Zaid and Dahman [34] in 1992 dealt by optimized design of prestressing force cross-section dimensions of simply supported post-tensioned beam with different cross-sections (rectangle, I and T section). They compared two objective function as weight of structures (concrete and prestressing reinforcement) with more objective cost function (concrete, prestressing reinforcement and formwork) by methods of linear programming. Constraints were normal concrete stresses, deflections, ultimate bending moment capacity and also buckling verification for very slender beams. The graph of dependencies between width of cross-sections and on objective functions. Both objective functions are possible to compare to limit length of the beam.

Cohn and Lounis [26] in 1993 presented an article about optimal design of partially and fully prestressed continuous beam. Ultimate and serviceability limit state condition were used as constraints in this study. Lagrange coefficient optimization method was used for optimized design of total cost of the structure (concrete, nonprestressed and prestressing reinforcement). As the conclusion of this work, the total costs of the structures were decreased by allowing of tensile concrete stress in concrete.

Khaleel and Itani [56] in 1993 studied optimized design of simply supported post-tensioned beam with I cross-section according to ACI 1983 code. Theirs objective function included cost for concrete, nonprestressed (longitudinal and stirrups) prestressing reinforcement. Sequential quadratic programming methods was used for solving of this problem. They solved structure in two states. The fist state was uncracked structure with normal concrete stresses in service and ultimate bending moment capacity as constraint. The second state is fully cracked structure where constraints were also ultimate bending moment capacity together with fatigue check. They concluded more economic is using of high strength prestressing material than allow tensile stress in concrete.

Lounis and Cohn [78] in 1995 investigated using of hollow section slab and box cross-section on prestressed bridge girder. Hollow section slab are more economic than prestressed box girder for span length < 20 m and width > 12 m and opposite for box girder. Another extension of theirs work was multi-objective analysis of such type of structure [76]. The results of this study were recommendations of maximal criteria for design and selection of bets concept of the bridge.

Al-Gahtani, Al-Sadoun and Abul-Feilat [12] in 1995 formulated effective tool for optimization of post-tensioned two span bridge girder according to ACI code. Costs for concrete, nonprestressed and prestressing reinforcement together with formwork were considered in objective function. Six groups of constraint were used for optimized design (geometrical, normal stresses of concrete, ultimate bending moment capacity, shear capacity detailing provisions and requirements for partially prestressed concrete). They again concluded the partially prestressed concrete is more economic than fully prestressed.

Han, Adamu and Karihaloo [45] in 1995 performed comparison of costs on partially prestressed rectangular beam with T beam according to Australian code using method DCOC (Discretization Continuum-type Optimality Criteria). The objective function concerned with cost for concrete, nonprestressed and prestressing reinforcement and formwork. A result of this work was the using of T section with partially prestressed concrete is more economic in simply supported beam. On year later [46] they minimized cost for continuous beams with two, three and four spans. DCOC method was used again in this case.

In 1997 Kirsch [62] returned to optimization of prestressed concrete structures by the optimization of post-tensioned continuous beam in two levels. The objective function included cost for concrete and prestressing reinforcement. The first level of optimization dealt with optimal design of prestressing force and geometry using process oriented decomposition method. The optimal dimensions of cross-section were optimized in the next step. Constraints for this case were normal stresses of concrete, deflections, geometrical constraints of tendon geometry and cross-section. He divided general complex problem to individual partial simplified subproblems and achieved more suitable and economic solutions.

Liu, Hammad and Itoh [75] in 1997 published work related to optimized design from the point of life-cycle, maintenance, regular checks cost of bridge deck system. They optimized costs for all mentioned cost before together with average deteoriation degree on six bridge structures in Nagoya City. Genetic algorithms were used for optimization process and they found group of the Pareto-optimal solutions of dependencies of cost and deteoriation degree.

Ohkubo, Dissanayke and Taniwaki [94] in 1998 considered multiobjective optimization design of the whole structural system of post-tensioned three span box bridge girders from minimum cost and maximum aesthetic behaviour of structure. They optimized span length with height of cross-section which has arbitrary cross-section. Normal stresses of concrete, crack resistance, ultimate bending moment capacity and ductility check according to American Building code 1990. Superstructure objective function consists of cost of concrete and prestressing reinforcement. Theory of fuzziness was used for optimization as very rare approach. Cost for piers and pile foundation were considered to substructure optimization. Aesthetic felling was expressed by the graphs of height of cross-section and span ratios.

Barakat, Kallas a Taha [18] published in 2003 an article about single objective optimized design of post-tensioned concrete beam based on the reliability. The reliability stochastic methods covered certain level of inaccuracies in modelling of material, load and others. These methods are more difficult, because they have to include optimization algorithm together with algorithm solving the reliability of the structure. Objective function included cost for concrete, nonprestressed and prestressed reinforcement and formwork. The optimization was performed on ultimate and serviceability limit states according to code ACI318-99. Conclusion of this study is designer approach. The first level of this approach is optimizing structure on serviceability limit states, modifies the inputs if needed and run optimization on ultimate limit states. Optimization is based on minimization of total cost with respect of the reliability of the constraints (probability of failure and reliability index). The optimized procedure was applied on 16 examples (4 different length and loads) of prestressed beams. The results of this study is optimal initial ratio equal to 10.5 (L/H) for I bridge girder.

One year after Barakat et al. [17] extended single objective optimization problem by bi and tri-objective optimization based on stochastic methods. Objective functions were minimization of total cost of prestressed beam, maximization of reliability index. Design variables were geometrical parameters of cross-sections, value and eccentricities of prestressing reinforcement. So called epsilon method was used for the solution of optimization problem. This method spread multiobjective problem to individual single objective subproblems. Pareto optimal solutions obtained by this multiobjective optimization method are more economic and effective than solutions from single objective.

Rana, Ahsan and Ghani [101] presented in 2010 an article about optimized design of post-tensioned concrete I beams. Their objective function included cost of concrete, nonprestressed and prestressed reinforcement, cost for manufacturing and final placement in structure. Design variables were dimensions of cross-sections, parameters of prestressing reinforcement, number and spacing of beams in transverse directions. Constraints were selected according to AASHTO. Evolution algorithm was used for optimization of this example. The optimized structure was Teesta Bridge in Bangladesh. It is 50 m long span bridge from prefabricated beam with post-tensioned tendons and cast-in-place deck. They decrease cost of structure of 35% using optimization algorithm.

Martí a González-Vidosa [84] published study in 2010 focused on optimal design of prefabricated prestressed foot bridge with 40 m long span and 6 m wide deck. Heuristic method of simulate annealing was used for optimization of this structure. Objective function consists of cost for concrete, nonprestressed and prestressing reinforcement. Constraints were ultimate bending moment capacity; shear capacity and deflection check according to Spanish EHE code. The investigated example had 59 design variables related to prestressing reinforcement geometry and geometry of the foot bridge cross-section. They concluded the method of simulate annealing is suitable for design of such kind of the structure.

### 2.6.2 Prestressed columns

Much less articles have been published related to prestressed column. Prestressed columns are usually statically determined structures and the calculation seems to be very easy on first view but complexity and nonlinearity constraint very frequently appear in this case.

One of the first published articles on optimization of prestressed column was study of Thakkar and Bulsari [117] in 1972. They optimized electric mast from pre-tensioned concrete according to Indian code. The shape of column was conic with Vierendel type. Cost for concrete and prestressing strands was included in the objective function. Optimization was based on analytical solution of constraints and design variables. They saved 18% of cost using this optimization algorithm.

Kocer a Arora [64] formulates optimized design of prestressed skimmed concrete poles. Two methods were used for optimization. The first method was branch and bound method for discrete design variables together with enumeration and sequential quadratic programming for continuous deign variables. The second one was genetic algorithm for both types of design variables. Total cost for manufacturing (concrete, prestressing wires and spirals) was selected as objective function. Requirements according to ACI318-77 (ultimate bending capacity, crack resistance, detailing provisions) were constraints. They reached saving about 25% related to original design.

Laníková and Štěpánek [69], [70] presented optimized design of prestressed skimmed concrete. They investigated nonprestressed and prestressed poles according to ČSN EN 1992-1-1. Amount of nonprestressed and prestressed reinforcement along pole length was objective function. Ultimate capacity of normal forces and bending moments where calculated strains were compared with limits was considered as constraints. Deterministic optimization was based on the repeating cycles (parametric study). The results of this work are the graphs of dependency between stress in prestressing reinforcement with total cost of the structure, deflection and crack width.

### 2.6.3 Prestressed slabs

Prestressed slabs is the another groups of structural members which deserves several commented articles. We can investigate design of prestressing and the whole structural system in this kind of structures from the point of minimization of cost as well. The design of prestressing could be difficult because there is two dimensional behaviour of the structure towards to one dimensional for beams. Advantage of the slab solution is the cross-section which is usually rectangle part with 1.0 m width of the whole slab.

The first found work related to optimized design of prestressed slab is work performed by Kirsch [61] in 1973. Load balanced method is used for design of prestressing force and problem was formulated like nonlinear programming.

Naaman [90] in 1976 compared minimal cost with minimal weight of the simply supported prestressed beam and one-way prestressed slab according the ACI code. Both objective function give the same results until ratio of cost of concrete/cost of reinforcement higher than 40 for pre-tensioned concrete. Ratio is equal 30 for post-tensioned concrete.

Rammamurthy [100] used general method of linear programming for optimal design of prestressed slab based on ultimate limit state constraints.

In 1987, MacRae and Cohn [80] performed optimization of prestressed flat slab according to Canadian standard using conjugant direct method. Only longitudinal reinforcement was considered during optimized design even if shear reinforcement could be significant. Optimization considered minimum cost for concrete and amount of nonprestressed and prestressed reinforcement. Several studies of different span, depth of the slab material characteristics, geometry and distribution of the tendons were calculated. They concluded very suitable is to concentrate strands to groups and use high strength steel for prestressing strands.

Kuycular [67] in 1991 presented work about optimized design of prestressed concrete slabs using load-balancing method. He found using uniform distribution of the tendons along the slab can be reached by the decreasing of cost about 20-30%.

Lounis and Cohn tried to optimized structure of prestressed slab based on minimal cost and maximal initial camber. Ultimate and serviceability limit states check according to ACI Building 1989 were considered as constraints. Finally, initial camber was transfer on constraints side. Methods of Lagrange coefficient was used for optimization procedure.

Krauser [66] described in his work optimal design of two-way post-tensioned concrete floor slab prestressed. Three different methods were used for parametric study. The first method was based on generation of plastic hinges. The second method was equivalent frame method and the last method was very well known load-balancing method.

### 2.6.4 Prestressed bridges

Many authors focused on optimization of prestressed concrete structures. The bridges are often one of the most expensive types of structures. Therefore, the importance of the cost decreasing is significant. Three tables bring the summaries of studied papers from several points of view. The first table (Tab. 2.1) compared the paper according to optimization method, number and types of parameters, calculation model and optimized part of structure. The different terms of objective functions are mentioned in the second table (Tab. 2.2). Last table is denoted to constraints with respect to used code, see Tab. 2.3.

Tab. 2.1 Members of the objective functions

| Authors | Year | PresType | ObjType | CON | PRE | LONG | STIR | FORM | DES | CONS |
|---|---|---|---|---|---|---|---|---|---|---|
| Naaman [89] | 1972 | Preten | Single | | | | | | | |
| Fereig [40] | 1985 | Preten | Single | ✓ | ✓ | ✓ | ✓ | | | ✓ |
| Yu, Gupta, Paul [124] | 1986 | Posten | Single | ✓ | ✓ | | | ✓ | | ✓ |
| Fereig, Smith [35] | 1990 | Posten | Param | | | | | | | |
| Marks, Trochymiak [83] | 1991 | Posten | Single | | ✓ | | | | | |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Quiroga Arroyo [99] | 1991 | Posten | Single | | ✓ | | | | | |
| Lounis, Cohn [79] | 1993 | Preten | Multi (cost + initial camber) | ✓ | ✓ | ✓ | ✓ | | | ✓ |
| Lounis, Cohn [77] | 1993 | Preten | Multi (cost + initial camber) | ✓ | ✓ | ✓ | ✓ | | | |
| Fereig [38] | 1994 | Preten | Single | ✓ | ✓ | ✓ | ✓ | | | |
| Lounis, Cohn [76] | 1995 | Preten | Multi (cost + total depth deck) | ✓ | ✓ | | | | | |
| Lounis, Cohn [78] | 1995 | Preten | Multi (cost + total depth deck) | ✓ | ✓ | | | | | |
| Hassanain [4] | 1998 | Preten | Single | ✓ | ✓ | ✓ | ✓ | | | ✓ |
| Ohkubo et al. [94] | 1998 | Posten | Multi (cost + aesthetic feeling) | ✓ | ✓ | | | | | |
| Hassanain, Loov [48] | 1999 | Preten | Single | ✓ | ✓ | ✓ | ✓ | | | ✓ |
| Sirca, Adeli [110] | 2005 | Preten | Single | ✓ | ✓ | ✓ | ✓ | ✓ | | ✓ |
| Sung, Chang, Teo [116] | 2006 | Posten | Single | ✓ | ✓ | ✓ | ✓ | | | |
| Aydın, Ayvaz [16] | 2009 | Preten | Single | ✓ | ✓ | | | | | |
| Martí, González-Vidosa [84] | 2010 | Preten | Single | ✓ | ✓ | ✓ | ✓ | | | |
| Rana, Ahsan, Ghani [101] | 2010 | Posten | Single | ✓ | ✓ | ✓ | ✓ | | | ✓ |
| Dlouhý, Novák [32] | 2010 | Posten | Single | | ✓ | | | | | |

Explanation for Tab. 2.1 - PresType – type of prestressing (Preten – pre-tensioned, Posten – post-tensioned); ObjType – number of objective function CON – concrete; PRES

– prestressing reinforcement; LONG – non-prestressed longitudinal reinforcement; STIR – shear reinforcement; FORM – formwork; DES – design costs; CONS – construction costs;

Tab. 2.2 Properties of optimization

| Authors | NoVar | TypeVar | OptiMeth | CSS | PresGeo | PresNo | OptiCSS |
|---|---|---|---|---|---|---|---|
| Naaman [89] | From library | List | Param. study | I section | ✓ | ✓ | ✓ |
| Fereig [40] | 10 | Cont. and list | Param. study | I section | ✓ | | ✓ |
| Yu, Gupta, Paul [124] | 5 | Discr. and cont. | Geom. program. | Box | ✓ | ✓ | ✓ |
| Fereig, Smith [35] | 10 | Cont. and list | Param. study | Box, hollow | ✓ | | From library |
| Marks, Trochymiak [83] | 47 | Cont. | Lin. program. (branch and bound) | Box | ✓ | ✓ | |
| Quiroga Arroyo [99] | 13 | Cont. | Gradient | Solid | ✓ | ✓ | |
| Lounis, Cohn [79] | 5 | Discr. and cont. | Lagrange coefficients, sieve-search | Phased I | ✓ | ✓ | ✓ |
| Lounis, Cohn [77] | 14 | Discr. and cont. | projected Lagrange method | I section | ✓ | ✓ | ✓ |
| Fereig [38] | 5 | Discrete | Param. study | Box | | ✓ | Only depth |
| Lounis, Cohn [76] | 6 | Cont. | projected Lagrange method | I, box, hollow core, parapet | ✓ | ✓ | ✓ |
| Lounis, Cohn [78] | 6 | Cont. | projected Lagrange method | I, box, hollow core, parapet | ✓ | ✓ | ✓ |
| Hassanain [4] | 8 | Cont. | Param. study | I section | ✓ | ✓ | From library |
| Ohkubo et al. [94] | 15 | Discr. and cont. | Theory of fuzziness | Box | ✓ | ✓ | ✓ |
| Hassanain, Loov [48] | 8 | Cont. | Param. study | I section | ✓ | ✓ | From library |
| Sirca, Adeli [110] | 7 | Discr. and cont. | Neural dynamic method | I section | ✓ | ✓ | From library |
| Sung, Chang, Teo [116] | 32 | Cont. | Minimum strain energy | Box | ✓ | | ✓ |
| Aydın, Ayvaz [16] | 9 | Discr. and cont. | Genetic algorithm | I section | ✓ | ✓ | ✓ |
| Martí, | 59 | Discr. and cont. | Modified simulated | Box | ✓ | ✓ | ✓ |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| González-Vidosa [84] | | | annealing | | | | |
| Rana, Ahsan, Ghani [101] | 14 | Discr. and cont. | Evolution algorithm | I section | ✓ | ✓ | ✓ |
| Dlouhý, Novák [32] | 8 | Discr. and cont. | Modified simulated annealing | Trapezoid | ✓ | ✓ | |

Explanation for Tab. 2.2 - NoVar – number of variables; TypeVar – variable types; OptiMeth – optimization method; CSS – used cross-section; PresGeo – optimization of prestressing geometry; PresNo – optimization of numbers of prestressing strands; OptiCss – cross-section optimization.

Tab. 2.3 Constraints

| Authors | Code | ACS | APS | ULS (N+M) | Shear | Crack | Def | Det | Css | PressEcc |
|---|---|---|---|---|---|---|---|---|---|---|
| Naaman [89] | AASHTO and CAN/CSA S6 | ✓ | ✓ | ✓ | ✓ | | | | ✓ | ✓ |
| Fereig [40] | CAN/CSA S6 | ✓ | | ✓ | | | | | | ✓ |
| Yu, Gupta, Paul [124] | BS | ✓ | | | ✓ | | | | ✓ | ✓ |
| Fereig, Smith [35] | CAN/CSA S6 | ✓ | | ✓ | | | | | | ✓ |
| Marks, Trochymiak [83] | PN | ✓ | | | | | | | | ✓ |
| Quiroga Arroyo [99] | Code independent | ✓ | | | | | | | | ✓ |
| Lounis, Cohn [79] | ACI | ✓ | ✓ | ✓ | ✓ | | ✓ | | ✓ | ✓ |
| Lounis, Cohn [77] | AASHTO | ✓ | | ✓ | ✓ | ✓ | ✓ | | ✓ | ✓ |
| Fereig [38] | AASHTO | ✓ | | ✓ | | | | | | ✓ |
| Lounis, Cohn [76] | OHBDC | ✓ | ✓ | ✓ | ✓ | ✓ | | | ✓ | ✓ |
| Lounis, Cohn [78] | OHBDC | ✓ | ✓ | ✓ | ✓ | ✓ | | | ✓ | ✓ |
| Hassanain [4] | OHBDC | ✓ | | ✓ | ✓ | | | ✓ | ✓ | ✓ |
| Ohkubo et al. [94] | ACI | ✓ | | ✓ | | ✓ | | ✓ | | |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Hassanain, Loov [48] | OHBDC | ✓ | | ✓ | ✓ | | | | ✓ | ✓ |
| Sirca, Adeli [110] | AASHTO | | | ✓ | ✓ | ✓ | | ✓ | ✓ | ✓ |
| Sung, Chang, Teo [116] | Code independent | ✓ | | | | | | | ✓ | |
| Aydın, Ayvaz [16] | AASHTO | ✓ | | ✓ | ✓ | | | | ✓ | ✓ |
| Martí, González-Vidosa [84] | EHE | | | ✓ | ✓ | | ✓ | ✓ | ✓ | ✓ |
| Rana, Ahsan, Ghani [101] | AASHTO | ✓ | | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Dlouhý, Novák [32] | EN 1992-2 | ✓ | ✓ | ✓ | | | | | | ✓ |

Explanation for Tab. 2.3 - Code – check code; ACS – allowable concrete stresses; APS – allowable stresses of prestressing reinforcement; ULS(N+M) – check of normal forces and bending in ultimate limit state; Shear – shear check; Crack – check of crack resistance; Def – deflection check; Det – detailing provisions check; Css – geometry of cross-section; PresEcc – eccentricity of prestressing reinforcement; * - buckling check was considered in addition.

### 2.6.5 Optimization of special types of prestressed structures

As the previous chapters indicate, we commented published papers about optimization of prestressed beams, columns and slabs. This section is reserved to optimization of special kind of prestressed structure like tubes, sleepers or structural details of structure.

The first example of special structure type is optimization of prestressed water tubes by Thakkar and Sridhar Rao [118] in 1974. The optimal dimensions and values of prestressing forces of tubes designed using simplex method of linear programming according to Indian code. They successfully applied this method on 230 km long tube from Veeranam Lake to Madras. The authors selected objective function as cost of concrete and prestressing reinforcement. The optimization was made on several construction stages (stage before and after transfer of longitudinal prestressing and before prestressing of wire wound structure, construction loads and the first crack appearance). They decreased cost of prestressing reinforcement about 38% using optimization which was about 4690 tons of prestressing reinforcement on 230 km long tubes.

Optimization of tensioned members according to ACI code performed Naaman [91] in 1982. Prestressed tensioned member are used in arch bridges, tanks or anchors. He applied method of nonlinear programming for design of arch bridge structure based on minimization of concrete and prestressing reinforcement cost of deck. Optimized design was drawn in graphs with dependency between area of prestressing reinforcement and concrete.

Optimization of anchorage zone of post-tensioned I beam was topic of work investigated by Zhongguo, Saleh and Tadros [126] in 1999. They used strut-and-tie model for design of additional transverse reinforcement in anchorage zone. Designed reinforcement transfer transversal tensile forces from prestressing and also shear force from external load. Optimization was performed using parametric study for selected typical cross-sections and value of prestressing force according to requirements of LRFD. The results were also compared with tests.

Formulation of prestressed concrete optimization of curved surface was topic of work made by Ohsaki and Fujiwara [95] in 2003. Curved surface are membranes consisted of small planes elements. The main problem of membrane structure is finding self-equilibrium state, which means the membrane has axial forces in its plane only. Form-finding method is used for optimization of this kind of structure. They used local and global formulation of membrane structure and concluded that local formulation is more accurate than global. Nevertheless, small curved membrane needs global formulation of problem.

Topology optimization of truss girders structure with free post-tensioned tendons was aim of Diaz and Mukherjee [31] in 2005. This kind of structure is still more popular for transparency, low selfweight and architectonic reasons. Dynamic vibration and Eigen values analysis are required for these structures. Effect of modal disparity [51] appears by application of external forces from free tendon to truss girders. Topology optimization deals where make localization of free tendon on structure to be its amount minimum together maximal possible modal disparity. In fact, it is topology optimization of truss girder structure. They used methods based on gradient of objective function and genetic algorithm.

Weiher et al. [121], in 2007, studied optimal distribution of the strands in the unbonded tendons inside PE prestressing duct. Interaction between tendon and duct can be expressed using co called strand factor. This coefficient gives stress of tendon with dependency on distribution of tendons in the duct. Transverse pressure in tendon is dangerous from the fatigue behaviour. The distribution of the tendon inside the duct is modelled using strut-and tie model in transverse direction. The distance between individual tendons were selected as objective function. They concluded the minimal suitable filling of the duct is between 40-50% of the duct.

In 2010, Song and Kong [111] optimized design of prestressed box girder exposed to chloride environment. Effect of chloride causes degradation of area of prestressing reinforcement and decreasing of prestressing force. They found decreasing of 30% of area of prestressing reinforcement in 50 years by the chloride. Total cost of structure (concrete, nonprestressed and prestressing reinforcement) was selected as objective function. Normal stresses in concrete in transfer and service, ultimate bending moment capacity, shear capacity and torsion capacity together with deflection check were considered as constraint according to AASHTO LRFD code. MATLAB functions with reliability factor were used for the optimization.

Optimization of cable-stayed bridge with parabolic pylon by Sung, Chang and Teo [116] in 2006 in Taiwan can be considered as special structure due to its complexity. Selfweight of two spans (119 + 59 m) bridge deck is suspended by 36 cables in 18 pairs. The aim of this study was to find optimal values of prestressing force in cables together with minimal cost for structure. This structure highly hyperstatic and they solve such structure using principles of minimum strain energy which find the optimal structure which has the minimum deformation. They succeed with optimization and decrease displacement of top nodes of pylon.

The last group of special structures is optimization of railway sleepers. Sadeghi and Babaee [106] studied optimal dimensions and prestressing wires in Sleeper B70. Five constraints were used for design (minimal and maximal pressure of ballast, vertical distance of sleepers form upper level of ballast, maximal and minimal bending moment in sleepers and their differences. They created a parametric study applied on 40 sleepers and compared with sleeper B70. The modified dimensions and shape of sleeper’s geometry and decrease cost by 21% and increase the resistance. Lutch [6] also investigated optimal design of prestressed concrete sleepers according to ACI code.

## 2.7 Conclusions

As follows from the previous chapters, the most of the optimization task related to prestressing dealt with pre-tensioned beams. Most authors focused on the prefabricated pre-tensioned girders. This process can be efficiently used during production in factory. A saving of little material of reinforcement bars in one girder can bring huge saving in overall merit. Several works concerned the optimization of the whole bridge system consisting of prefabricated members. Nevertheless, the optimization of the post-tensioned bridges is only commented in various articles. There are several reasons for that. Concrete structures are very complex accompanied with many discrete variables, dozens of constraints required by codes and several types of nonlinearities. Additionally, the geometry of the tendon is not straight as is usual for pre-tensioned beams. The post-tensioned structures are very sensitive to changes of any parameters. An active role of prestressing represents the main possibility of optimization. Post-tensioned structures often require a deep analysis usually including the respect of the construction stages and rheological influence. The preparation of algorithm calculating post-tensioned structures with all mentioned effects is not a simple task. Additionally, the optimization process has to be covered during repeating cycle.

Consequently, the thesis focuses on the optimization of the post-tensioned concrete structures, mainly the geometry of the post-tensioned tendon. A special concern for covering the time dependent effect in the optimization task is taken care of. The cable structures are a special group of prestressed structures which are always highly statically indeterminate. Therefore, another consideration is let on the optimization of cable forces in the cable-stayed structures.

As already said, optimization of prestressed concrete structures can be seen from different explications of the objective function. The weight of the structure is one of the main criteria. Post-tensioned structures have usually fixed dimensions. Therefore, we focus mainly on the optimization task based on the area and weight of the post-tensioned tendons. A more objective view is to perform cost optimization which covers the cost of the whole construction.

# 3 OBJECTIVES

The objective of this study is to bring new possibilities for the design and check of the prestressed concrete structures. Design is currently based on the engineer’s experience, their knowledge and information on existing structures.

Generally, there are defined three main objectives of this thesis:

1. The first main goal is a studying of several kinds of optimization algorithms and theirs implementation to general procedure of optimal design of structures. The objective is reached through a program combining a structural finite element analysis with optimization techniques.
2. The second main target is a verification of proposed method for simple prestressed concrete structure. However, the proposed procedure is applicable to any kind of the structure as we focus on the design of prestressed structures using several criteria within one step. The testing concludes, which optimization algorithms are the most suitable for optimization of prestressed concrete structures with their advantages and disadvantages.
3. The last main goal is the application of proposed optimization procedure with the most suitable optimization algorithm to several real examples made from prestressed concrete. This thesis studies the use of the optimization of different types of the prestressed structures as prestressed concrete 1D element bridges, prestressed concrete slabs and cable-stayed structures. Furthermore, the finding of the most efficient constraints belongs to important factor during optimization. A structural reliability and final cost reduction is another significant aspect which is verified. Finally, this thesis provides a recommendation to design of prestressed concrete structures.

# 4 OPTIMIZATION ALGORITHMS

An optimization problem can be solved using two different ways. The first is the analytical way where the solution can be found very fast for a simple structure. The problem usually is with complicated structures requiring the fulfilment of many code constraints. Therefore, the second group of optimal design is inevitably applied. The classification of optimization method can be seen from different points of views like criteria, requirements, optimization process, etc. One of frequent divisions is based on the type of used values. Deterministic methods are managed by the results usually obtained from linear programming technique. The methods are very efficient due to a small number of iteration although they are usable for a small number of parameters only. The main disadvantage of the deterministic method is insufficient work with discrete parameters. Stochastic methods, which are the second type, allow for random behaviour of values. These successfully find the global extreme instead of local one. The combination of both approaches brings the most efficient optimization algorithm. Habiballa [4] sorted typical optimization methods according to three basic types mentioned in Tab. 4.1.

Tab. 4.1 Three types of optimization algorithm [4]

| Deterministic | Stochastic | Mixed |
|---|---|---|
| Hill-climbing [104] | Simulated annealing [29] | Ant colony optimization [28] |
| Branch and bound [68] | Monte Carlo [88] | Memetic algorithm [30] |
| Greedy [19] | Tabu search [42] | Genetic algorithms [82] |
| Calculus based | Latin hypercube sampling [9] | Particle swarm [55] |

As we seen, there exist dozen optimization techniques but only several can be used for the optimization of prestressed concrete structures. There are many aspects of usability as the complexity of optimization tasks, lot of constraints in civil engineering design codes and parameters of buildings, bridges and special type structures are far from “smooth”. Used optimization method in this thesis can be divided into three basic groups:

- <u>Evolutionary algorithm</u> (see Chapter 4.1) – differential evolution, modified simulated annealing
- <u>Gradient based method</u> (see Chapter 4.2) – sequential quadratic programming
- <u>Heuristic method</u> (see Chapter 4.3) – simplex method (Nelder-Mead method)

## 4.1 Evolutionary algorithm

Evolutionary algorithms are very robust simulating evolution process in nature. There are usable for wide range of optimization problems with low sensitivity to errors. The main principle is that any weak member of population cannot be presented in new generation. Possibly methods are used for optimization of standard functions with several local extremes. Large numbers of iterations and small inaccuracy from optimum can be the disadvantages of this method.

The used terminology

- *member* – vector of $D$ + 1 items; $J^{i,j} = [x^{i,j}_{k=1}, x^{i,j}_{k=2}, f^{i,j}]$
- *population* – group of $i$ members $i = 1,\dots NP$
- *generation* – $j$ population $j = 0,\dots G$
- *evolution* – sequence of $G$ generations $k = 1,\dots D$

### 4.1.1 Differential evolution (DE)

The differential evolution is stochastic searching optimization algorithm. Method was established by Storn and Price [114], [115] in 1995. Generally, the method is very robust and relatively fast converging also for multidimensional problems. A group of more optimum solutions is usually resulting from this method.

The number of vectors is randomly selected from population of possible solutions. New testable vectors $u_k^{i,j+1}$ are generated by combination of randomly selected vectors from current existing population $x_k^{i,j}$ in iteration. Differential evolution generates new vector based on calculation of weighted difference between two randomly selected members of population and this result is added to third member. When new vector has better value of objective function than initial one then new vector replaces initial vector.

An algorithm can be described in several points

- **Method parameter setting**

  A reproduction cycle is managed by the following settings

  - Number of member in population NP > 3
  - Mutation constant $F \in < \langle 0; 2 \rangle$
  - Crossing ratio $CR \in \langle 0; 1 \rangle$

- **First population**

  The first generation is made using random selection (rand) from the vectors of searched design variables $x$(min, max)

$$x_{\mathrm{k}}^{\mathrm{i},1} = x_{\mathrm{k,min}} + \mathrm{rand}\left(x_{\mathrm{k,max}} - x_{\mathrm{k,min}}\right). \tag{4.1}$$

- **Reproduction cycle - permutation**

  A tested vector is generated for each member of population. Recently three different members are selected form the same generation ($r_1$; $r_2$; $r_3$). A difference of objective function from the first two members is multiplied by the mutation constant $F$ and added to the third member. It is illustrated graphically in Fig. 4.1.

$$v_{\mathrm{k}}^{\mathrm{i,j+1}} = x_{\mathrm{k}}^{\mathrm{r3,j}} + \mathrm{F} \cdot \left(x_{\mathrm{k}}^{\mathrm{r1,j}} - x_{\mathrm{k}}^{\mathrm{r2j}}\right). \tag{4.2}$$

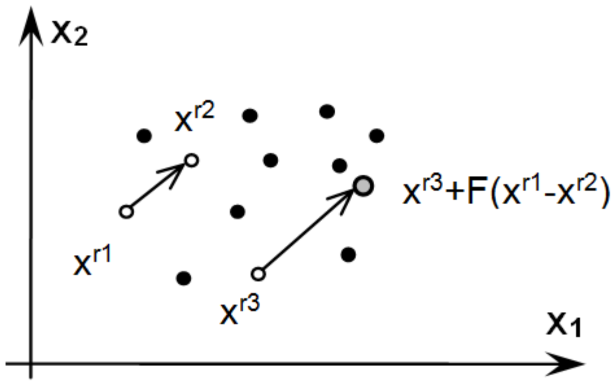



Fig. 4.1 Creation of tested vector

  Afterwards each member from population and from the tested vector is selected. The random number with uniform probability distribution in range (0-1) is generated for both members. The random number is compared with crossing constant and two cases can appear:

  - If the random number is less than the crossing ratio, the new vector $u_i$ is based on the initial vector $x_i$ (parent) $u_{i,k} = x_{i,k}$.
  - If the random number is higher than crossing ratio, the new vector $u_i$ is based on the tested vector $v_i$ (child) $u_{i,k} = v_{i,k}$.

- **Calculation of objective function for tested members**

  The objective function $f$ is calculated for each tested member.

- **Determination of new population**

  New generation is determined using comparing of objective function of initial and new tested vectors. Two types can occur in case of minimization of objective function:

  - If the objective function of tested vector is less than initial member, then new tested vector belongs to new generation

$$f(u_{\mathrm{i,j+1}}) \leq f(x_{\mathrm{i,j}}) \rightarrow J^{\mathrm{i,j+1}} = u^{\mathrm{i}}. \tag{4.3}$$

- If the objective function of tested vector is higher than initial member then the initial member stays in new generation

$$f(u_{i,j+1}) > f(x_{i,j}) \rightarrow J^{i,j+1} = J^{i}. \quad (4.4)$$

This behaviour ensures that any worse member than parents is a part if there is a new generation. Therefore population tends to the global extreme.

- **Evolution evaluation**

  The cycle from reproduction to determination of new population is repeated to convergence criteria. The best member or group of them are declared as optimal solution.

### 4.1.2 Modified simulated annealing (MSA)

Generally, this method is very robust with low error sensitivity. Relatively great number of iterations and little inaccuracy to optimal solution are small disadvantages. Method is based on combination of genetic algorithm and standard simulated annealing. Genetic algorithm selects certain member from population. Simulated annealing calculates acceptance probability of this member to population.

#### 4.1.2.1 Genetic algorithm

Originally the study of Holland on cellular automata was foundation of the genetic algorithm in 60 years of last century. Goldberg work [82] can be considered as the first genetic algorithm article 30 years ago. Genetic algorithm is heuristic procedure applying evolutionary principles from biology for solving of mathematical problems where exact method is not sufficient. A techniques simulating evolutionary principles as hereditary, mutation and crossing are used for step by step creation of new generation of various solution of problem. The population where each member represents one solution of problem is stored in population. The solution improves during evolution process. Typically the first generation is based on the randomly selected members. New generation requires calculation of so called fitness function describing quality of solved member. Stochastic selection is applied according to quality of the members. Each selected member is modified using mutation and crossing for new population. This procedure is iteratively repeated commonly with quality of the members. The algorithm stops after reached satisfied quality or exceeded number of iterations.

As genetic algorithm is an unconstrained algorithm, it is necessary to convert the constrained optimization task to unconstrained one. Several methods have been developed for this. One of the most used methods is the using of penalty function. This approach transform constrained problem to unconstrained one by adding penalty for each constraint exceeding to the objective function.

#### 4.1.2.2 Simulated annealing

Independently two group of engineers described simulated annealing method. The first group was Kirkpatrick, Gelatt and Vecchi [57] in 1983. Černý [29] represented the second group in 1985. The stochastic method is analogy to physical process of metal annealing. Continuous cooling causes energetic minimum of the material representing global minimum.

The method should ensure that each point of set of numbers is used at least once for calculation. Sometimes the algorithm can stay in local extreme. This behaviour is covered using wider steps of cooling temperature at the beginning and fewer steps at the end of the process. The step proportion is based on the temperature. High temperature causes the high changes. Generally more annealing methods exist. All existing types have the same questions:

- What are the most effective initial temperature and the best algorithm used for cooling?
- What is the best generation of the members tested using simulated annealing?

#### 4.1.2.3 Procedure of modified simulated annealing

The basic procedure of modified simulated annealing is described in this chapter. As was mentioned recently, it is combination of genetic algorithm and simulated annealing.

**1) Initiation**

The first population consists of randomly generated *n* members using KISS generator and the objective functions are calculated. The members are sorted according to its quality (fitness function)

$$\mathrm{fit}\left(x_{\mathrm{k}}^{1,\mathrm{j}}\right) = \left\{\mathrm{fit}\left(x_{\mathrm{k}}^{1,\mathrm{j}}\right)_{\mathrm{min}} \ldots \mathrm{fit}\left(x_{\mathrm{k}}^{1,\mathrm{j}}\right)_{\mathrm{n}/2} \ldots \mathrm{fit}\left(x_{\mathrm{k}}^{1,\mathrm{j}}\right)_{\mathrm{n=max}}\right\}. \tag{4.5}$$

**2) Method parameter setting**

The initial annealing temperature $T_0$ is selected according to 50% ratio of accepted to all members. This assumption is used for calculation of initial annealing temperature $T_0 = T_{\mathrm{max}}$.

$$p = \mathrm{e}^{\frac{-\Delta f}{T_0}} = 0.5 \rightarrow T_0 = T_{\mathrm{max}} = \frac{-\Delta f}{\ln(0.5)} = \frac{-\left\{f\left(x_{\mathrm{k}}^{1,\mathrm{j}}\right)_{\frac{\mathrm{n}}{2}} - f\left(x_{\mathrm{k}}^{1,\mathrm{j}}\right)_{\mathrm{min}}\right\}}{\ln(0.5)}. \tag{4.6}$$

The minimal annealing temperature ($T_{\mathrm{min}}$) is calculated according to ratio of maximal and minimal temperature $\left(c_{\mathrm{Tmax,min}}=\frac{T_{\mathrm{min}}}{T_{\mathrm{max}}}\right)$.

$$T_{\mathrm{min}} = c_{\mathrm{Tmax,min}} \cdot T_{\mathrm{max}}. \tag{4.7}$$

Furthermore, so called annealing constant ($T_{\mathrm{mult}}$) is determined expressing rational value of temperature decreasing during cooling. Number of iteration ($N_{\mathrm{iter}}$) is defined in the method setting.

$$T_{\mathrm{min}} = {T_{\mathrm{mult}}}^{N_{\mathrm{iter}}} \cdot T_{\mathrm{max}} \rightarrow T_{\mathrm{mult}} = \sqrt[N_{\mathrm{iter}}]{\frac{T_{\mathrm{min}}}{T_{\mathrm{max}}}}. \tag{4.8}$$

**3) Reproduction**

The crossing and mutation processes generate new members from the selected members and new population exists. Afterwards members are compared within initial members based on their objective function.

- If the objective function of new member is better than initial member, then new generation includes new member. In case of minimization of objective function

$$f(x_{\mathrm{i,j+1}}) \leq f(x_{\mathrm{i,j}}) \rightarrow J^{\mathrm{i,j+1}} = x_{\mathrm{i,j+1}}. \tag{4.9}$$

- If the objective function of new member is worse than initial member, then new generation includes initial member. In case of minimization of objective function

$$f(x_{\mathrm{i,j+1}}) > f(x_{\mathrm{i,j}}) \rightarrow J^{\mathrm{i,j+1}} = x_{\mathrm{i,j}}. \tag{4.10}$$

This procedure is running to finalizing iteration of particular temperature level. The iteration stops when defined number of better member ($succ_{\mathrm{max}}$) is reached or maximal defined number of all iteration ($count_{\mathrm{max}}$) is done even if number of better member ($succ_{\mathrm{max}}$) is not get.

- ***crossing*** – changing of several parts between members

  Example of crossing A and B on 5th position.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Number A - parent | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 |
| Number B - parent | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 |

$$(A'; B') = O_{\mathrm{cross}}(A; B). \tag{4.11}$$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Number A´ - child | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 |
| Number B´ - child | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |

- ***mutation*** – random change of some member part

  Example of crossing A on its 1st, 5th and 7th position.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Number A | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 |
| Number A´ | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |

$$(A') = O_{\mathrm{mut}}(A). \tag{4.12}$$

**4) Acceptance criteria – probability of acceptance**

Generally, a gradient based algorithm accepts new members in case of better objective function only. In case of simulated annealing the also worse solution are accepted with certain probability. The probability directly depends on the temperature; see Fig. 4.2.

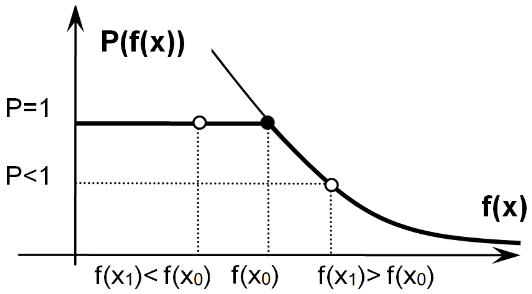



Fig. 4.2 Probability acceptance of member in case of the objective function minimization

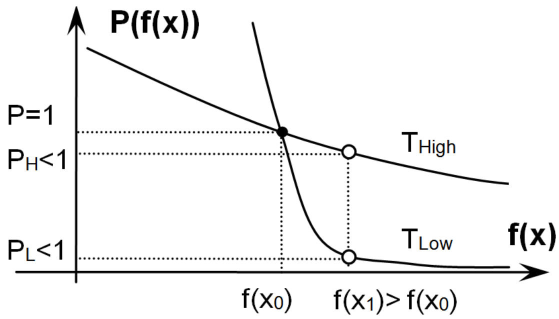



Fig. 4.3 Temperature dependency of the member acceptance probability

Temperature is continuously decreased based on speed of convergence. When the algorithm converge fast then the temperature decreases as well. If the method converges slowly, the decreasing of the temperature is also performed; see Fig. 4.3. Thus, local extreme achievement is neglected. Maximal number of iteration is usually stop criteria. The probability of the acceptance is calculated for each visited member. This can be expressed as the following

$$p = \mathrm{e}^{-\Delta f/T}. \tag{4.13}$$

where

*Δf* – energy difference between two states of crystal system, it means objective function difference between two selected members; $\Delta f = f(x_{k+1}) - f(x_k)$

*T* – annealing temperature

In case of minimization of objective function, the new member is accepted with a lower value of the objective function. The opposite case also allows for accepting of the member but with certain probability

$$p(x_\mathrm{k} \rightarrow x_\mathrm{k+1}) = \begin{cases} 1 & \text{pro } f(x_\mathrm{k+1}) \le f(x_\mathrm{k}) \\ p = \mathrm{e}^{-\frac{\Delta f}{T}} & \text{pro } f(x_\mathrm{k+1}) > f(x_\mathrm{k}) \end{cases}. \quad (4.14)$$

**5) Calculation of new annealing temperature - cooling**

As was mentioned before the temperature is decrease during the cooling process. When algorithm converges fast the temperature is decrease fast too. The previous temperature level is multiplied by the annealing constant in each cooling step

$$T_\mathrm{i+1} = T_\mathrm{mult} \cdot T_\mathrm{i}. \quad (4.15)$$

**6) Cycle repeating**

The process of cooling is performed according to distribution of interval between initial (maximal) and minimal temperature. The distribution depends on number of iteration $N_\mathrm{iter}$. The genetic algorithm applies crossing and mutation for each temperature level. The members from new populations are accepted based on the probability.

**7) Stop criteria**

The algorithm finishes reaching minimal annealing temperature. The best member or the set of best members are declared as the optimal solution.

## 4.2 Gradient based method – sequential quadratic programming

Generally, the method based on the gradient calculation requires a small amount of iterations. The convergence near the optimum is very fast. Nevertheless, one of the disadvantages is high sensitivity to optimized functions. Usually it fails for discrete type of objective function (e.g. concrete checks).

The method using gradient can be described in the following steps.

- Optimization start - select $x_0$
- Calculation of partial derivation

$$\nabla f = \frac{\partial f}{\delta x_\mathrm{i}} \text{ for } \mathrm{i} = 1, 2 \dots n. \quad (4.16)$$

- Calculation of new solution

$$x_\mathrm{i}^\mathrm{n} = x_\mathrm{i}^\mathrm{n-1} + \nabla f;\ \text{for i} = 1, 2 \dots n. \quad (4.17)$$

- Cycle 2 and 3 running to stop criteria (convergence criterion of accepted error ε)

$$\|\nabla f\| \le \varepsilon. \quad (4.18)$$

### 4.2.1 Sequential quadratic programming (SQP)

Sequential quadratic method is one of the methods using calculation of gradient of objective function. As stated before for gradient method, SQP method requires a small amount of iterations and it is relatively accurate algorithm. In comparison with the MSA method, the SQP method is less robust and usable for continuous design variables. This method can be known also as method of the biggest slope. The solving of the partial steps designed as minimization of quadratic model of the problem satisfying constraints is main idea of the method. When the constraints are not defined then SQP method is transformed to unconstrained classical Newton method.

The equation of objective function is solved in each step of iteration and the method is closer to optimum. SQP method uses quadratic approximation of objective function $f(x)$ and linear approximation of constraints $g(x)$. Optimization problem can be formulated as follows

$$\begin{aligned} &\min f(x) \\ &\text{s.t. } g_{\mathrm{k}}(x) \le 0;\ k = 1, 2 \dots p. \end{aligned} \tag{4.19}$$

In case of SQP method using gradient

$$f(x_{\mathrm{k}} + \Delta x) \approx f(x_{\mathrm{k}}) + \nabla f(x_{\mathrm{k}})^{\mathrm{T}} \Delta x. \tag{4.20}$$

Taylor polynomial replaces previous equation

$$f(x_{\mathrm{k}} + \Delta x) \approx f(x_{\mathrm{k}}) + \nabla f(x_{\mathrm{k}}) \Delta x + \frac{1}{2} \Delta x^{\mathrm{T}} \nabla^2 f(x_{\mathrm{k}}) \Delta x. \tag{4.21}$$

The first and second derivation can be substitute by Jacobi and Hessian matrix.

$$f(x_{\mathrm{k}} + \Delta x) \approx f(x_{\mathrm{k}}) + J(x_{\mathrm{k}}) \Delta x + \frac{1}{2} \Delta x^{\mathrm{T}} H(x_{\mathrm{k}}) \Delta x. \tag{4.22}$$

The Lagrange coefficients ($\lambda$) are used for the solving this problem

$$l(x, \lambda) = f(x), \tag{4.23}$$

$$l(x_{\mathrm{k}} + \Delta x, \lambda_{\mathrm{k}}) \approx l(x_{\mathrm{k}}, \lambda_{\mathrm{k}}) + \nabla l(x_{\mathrm{k}}, \lambda_{\mathrm{k}}) \Delta x + \frac{1}{2} \Delta x^{\mathrm{T}} \nabla^2 l(x_{\mathrm{k}}, \lambda_{\mathrm{k}}) \Delta x, \tag{4.24}$$

$$L(x, \lambda) = f(x) + \sum_{\mathrm{k}=1}^{\mathrm{p}} \lambda_{\mathrm{k}} \cdot g_{\mathrm{k}}(x). \tag{4.25}$$

Furthermore, Kuhn-Tucker condition for minimization (gradient $L = 0$) transforms formulas to

$$\nabla L = \nabla f(x) + \sum_{\mathrm{k}=1}^{\mathrm{p}} \lambda_{\mathrm{k}} \nabla g_{\mathrm{k}}(x) = 0, \tag{4.26}$$

$$l(x_{\mathrm{k}} + \Delta x, \lambda_{\mathrm{k}}) \approx l(x_{\mathrm{k}}, \lambda_{\mathrm{k}}) + J(x_{\mathrm{k}}, \lambda_{\mathrm{k}}) \Delta x + \frac{1}{2} \Delta x^{\mathrm{T}} H(x_{\mathrm{k}}, \lambda_{\mathrm{k}}) \Delta x. \tag{4.27}$$

Hessian matrix is not calculated in each step of SQP method but it is approximated using BFGS method (Broyden–Fletcher–Goldfarb–Shanno**)** from the previous step

$$H_{\mathrm{k+1}} = H_{\mathrm{k}} + \frac{y_{\mathrm{k}} y_{\mathrm{k}}^{\mathrm{T}}}{y_{\mathrm{k}}^{\mathrm{T}} \Delta x_{\mathrm{k}}} - \frac{H_{\mathrm{k}} \Delta x_{\mathrm{k}} (H_{\mathrm{k}} \Delta x_{\mathrm{k}})^{\mathrm{T}}}{\Delta x_{\mathrm{k}}^{\mathrm{T}} H_{\mathrm{k}} \Delta x_{\mathrm{k}}}, \tag{4.28}$$

where

$$y_{\mathrm{k}} = \nabla f(\Delta x_{\mathrm{k+1}}) - \nabla f(\Delta x_{\mathrm{k}}). \tag{4.29}$$

## 4.3 Heuristic Algorithm – simplex method (Nelder – Mead method)

Heuristic methods are algorithms with lower sensitivity to objective functions than gradient based method but they require more iteration. As the name indicates, the method was first described by Nelder and Mead [93]. This algorithm uses the concept of a simplex for minimizing of an objective function in a many-dimensional space. A sequence of triangles is generated during the method and functional values at the vertices get better and better. The size of the triangles is reduced and the coordinates of the extreme point are found. In general, the output of any objective function of *n* variables is evaluated by a set of ($n$ + 1) points of general simplex. The vertices with the highest value of objective function are replaced by another point. Thus, an extreme of that function is found by the transforming of one point of simplex in the space. The replacement can be performed by the four basic operations (i) reflection, (ii) contraction, (iii) expansion and (iv) reduction, see Fig. 4.4.

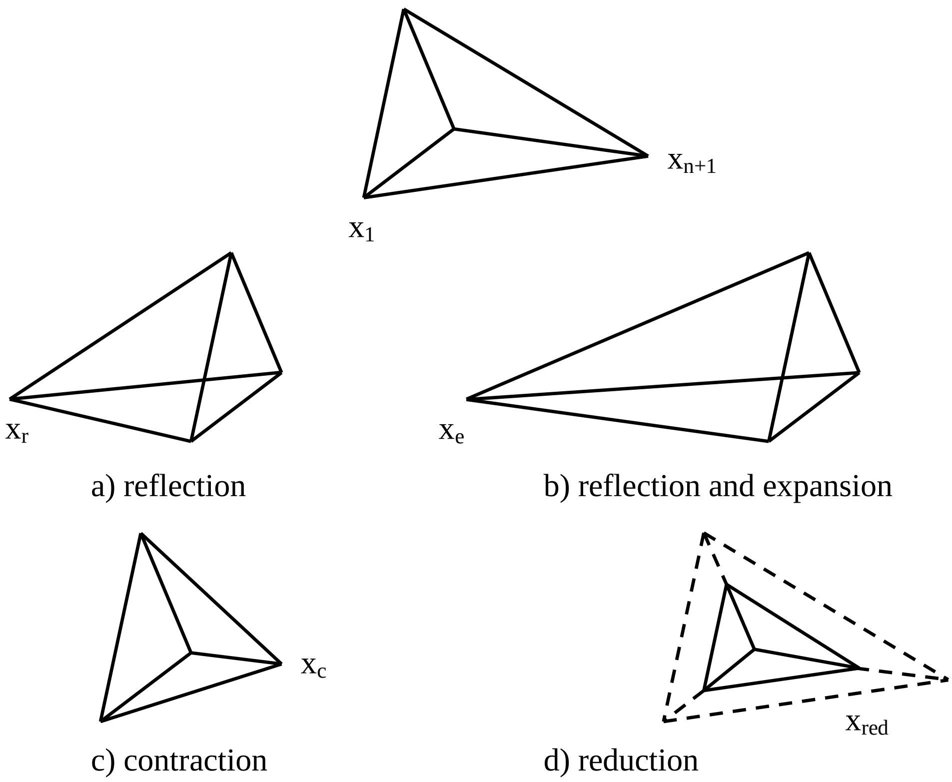



Fig. 4.4 Graphical presentation of particular operation during NM

Let us assume an objective function $f(x)$ with n variables in ($n$ +1) points of $n$-dimensional space defining the simplex. All vertices have to be sorted according to its values. Furthermore, the centre of the gravity of all simplex points (except $x_{n+1}$) is calculated as $x_0$. As has been mentioned above, three basic operations modify the simplex to global extreme.

### 4.3.1 Reflection

A new vertex of simplex is defined as follows

$$x_{\mathrm{r}} = x_0 + \alpha \cdot (x_0 - x_{\mathrm{n+1}}), \tag{4.30}$$

where α is the reflection coefficient $>0$ usually equal to 1. When the new reflected point has better objective function than the second worst, then replace the worst point by the reflected point in a new simplex. It can be written in the following terms

$$f(x_1) \leq f(x_{\mathrm{r}}) \leq f(x_{\mathrm{n}}). \tag{4.31}$$

### 4.3.2 Expansion

In case of new reflected point produces new minimum $\big(f(x_{\mathrm{r}}) \leq f(x_1)\big)$ then the expansion is defined as follows

$$x_{\mathrm{e}} = x_0 + \gamma \cdot (x_{\mathrm{r}} - x_0), \tag{4.32}$$

where γ is the reflection coefficient >1.0 usually equal to 2. When the new expanded point has better objective function than the reflected point $\big(f(x_{\mathrm{e}}) \leq f(x_{\mathrm{r}})\big)$, then the worst point $(x_{n+1})$ is replaced by the expanded point in a new simplex. Otherwise, the reflected point remains. If the reflected point has not better objective function, than second worst we can continue with contraction.

### 4.3.3 Contraction

If the reflected point has not better objective function than second worst $\big(f(x_{\mathrm{r}}) \geq f(x_{\mathrm{n}})\big)$, the algorithm continues with contraction expressed using following formula

$$x_{\mathrm{c}} = x_0 + \beta \cdot (x_{\mathrm{n+1}} - x_0), \tag{4.33}$$

where β is the contraction coefficient $\langle 0; 1 \rangle$ usually equal to 0.5. When the new contracted point has better objective function than then the worst point $\big(f(x_{\mathrm{c}}) \leq f(x_{\mathrm{n+1}})\big)$, then this point $(x_{n+1})$ is replaced by the contracted point $(x_{\mathrm{c}})$ in a new simplex. Otherwise, the reflected point remains.

### 4.3.4 Internal contraction (reduction)

The last possibility how to obtain extreme of function is using the internal contraction (reduction). In this case $\left(f(x_{\mathrm{c}}) > f(x_{\mathrm{n+1}})\right)$, new reduced point is written as follows

$$x_{\mathrm{red}} = x_1 + \sigma \cdot (x_{\mathrm{i}} - x_1), \tag{4.34}$$

where σ is the reduction coefficient usually equal to 0.5.

Generally, the algorithm terminates when the objective function is less than standard error $(\varepsilon)$ expressed with inequality

$$\sqrt{\sum_{\mathrm{i}=1}^{\mathrm{n}+1} \frac{\left(f(x_{\mathrm{i}}) - f(x_0)\right)^2}{n}} < \varepsilon. \tag{4.35}$$

Hereby, the better solution is not found by one of the operation or the simplex remains the same in particular cycle.

Simultaneously, the algorithm is able to find minimum when starting far from them. It concludes to relatively robust algorithm. The main disadvantage is usability only on a small numbers of parameters. As will be shown in the following text, the effectiveness of the method does not seem to be good for design of prestressed concrete structures. This method representing heuristic algorithm was used only for specimen in Chapter 6.

## 4.4 Comparison of optimization algorithms and conclusions

The previous chapters dealt with the description of the optimization algorithm. When we optimize the structure, each project is unique and requires a different view of the solution. We can see in Tab. 4.2 the comparison of used optimization algorithms by the several criteria which concluded from our verifications. There are two main criteria: (i) method characteristic and (ii) project characteristics. If we compare the convergence of the method the fast method is SQP. On the other hand, there is the very slowly convergent MSA method. Nevertheless, MSA is very robust method which is able to find the optimum in a wide space of design variables. Obviously, SQP can have a problem with searching of the global optimum. The usability of proper algorithm also depends on the project characteristics. Relatively small projects can be solved by the NM and SQP method. Genetic algorithms have no problem with the optimization of large structures. Sometimes, the design variables are even discrete (e.g. number or diameter of strands).

Tab. 4.2 Comparison of optimization algorithms

| Method | Method characteristics | | Project characteristics | | Typical examples |
|---|---|---|---|---|---|
| | Convergence | Robustness | Number of parameters | Type of parameters | |
| NM | medium | low | small | continuous | truss girder |
| SQP | fast | very low | medium | continuous | frame steel structures |
| DE | slow | medium | large | continuous + discrete | concrete structures |
| MSA | very slow | high | large | continuous + discrete | concrete structures, large projects |

We can also compare the convergence and number of iterations for a particular method in Fig. 4.5. It is clear from the graphical comparison that methods have specific ways how to find the optimum. This figure shows the optimization of each method from the initial to the optimal computational state. A different colour represents one design variable in the graph. In this illustrative example the SQP method was confirmed to be very fast, requiring tens of iterations only. As was expected, DE and MSA need hundreds of iterations to find the optimum. Consequently, it is clear the universal algorithm does not exist. There is no general rule that would always say in advance which method is the best for which type of optimisation task.

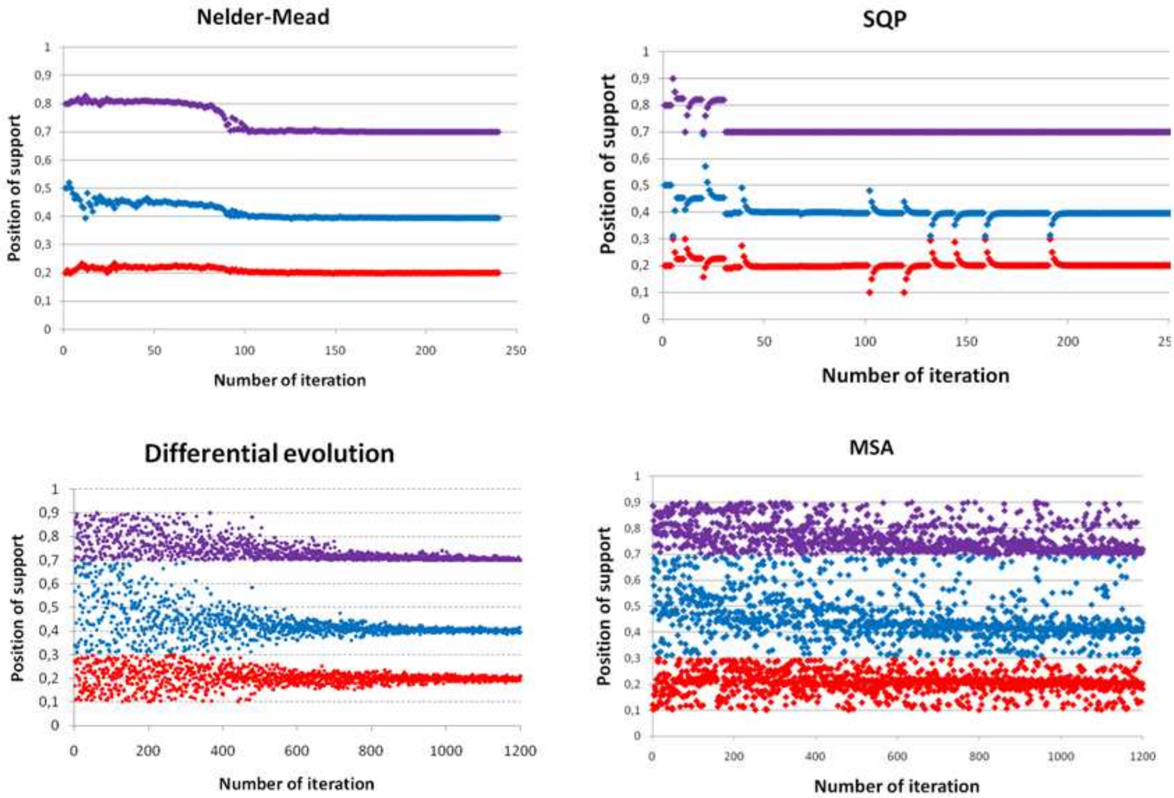



Fig. 4.5 Comparison of optimization methods

# 5 OPTIMIZATION PROCEDURE

The global optimization of structures is a difficult process requiring multiple recalculation of the whole structure with varying design parameters and evaluation of each configuration. The repetitive calculation of hundreds of solutions is usually not possible in daily practice. Designers are under time pressure and they finalize a project with several code satisfying solutions. Nowadays high performance computers are inevitably used for the design of structures. There exist many commercial computer programs for structural design and check. Some of them can also be used to test a wide range of configurations of parameter possibilities, to make comparisons and find the best solution. It is a question for the designer which optimization technique, constraints and limits for design variables are selected. The optimization is not something which completely replaces the designer. It is an additional tool for better effectiveness and performance of their work. An optimal solution can easily be found for simple structures with a very small number of design variables. The optimization of the large structures is dependent on the computer capacity, which is another important factor for the selection of optimization algorithm. The nighttime inactivity of computers can be effectively used for the optimization of structures. Before leaving work, the engineer prepares the optimization task and an optimum solution is found during the night at the latest. An algorithm finds one or several optimums. Afterwards the designer can select one from the array of offered solutions and verify the result manually if it really fits their requirements.

Currently, Nemetschek Scia is developing a special tool for optimization of any kind of structure. The working name is Scia Engineer Optimization Toolbox (hereinafter EOT). Sometimes, it can be found as Scia Optimizer [21]. Generally, the optimization process can be clearly seen in Fig. 5.1. Once all the required input data are entered, i.e. the model of the analysed structure is defined, the search for the optimal solution runs fully automatically and no interaction from the engineer is required. For real-life problems several optimal solutions can be found. In such situations, it is up to the engineer to make the final decision. More information can be found in [136]. The whole procedure of optimization can be explained in the following steps.

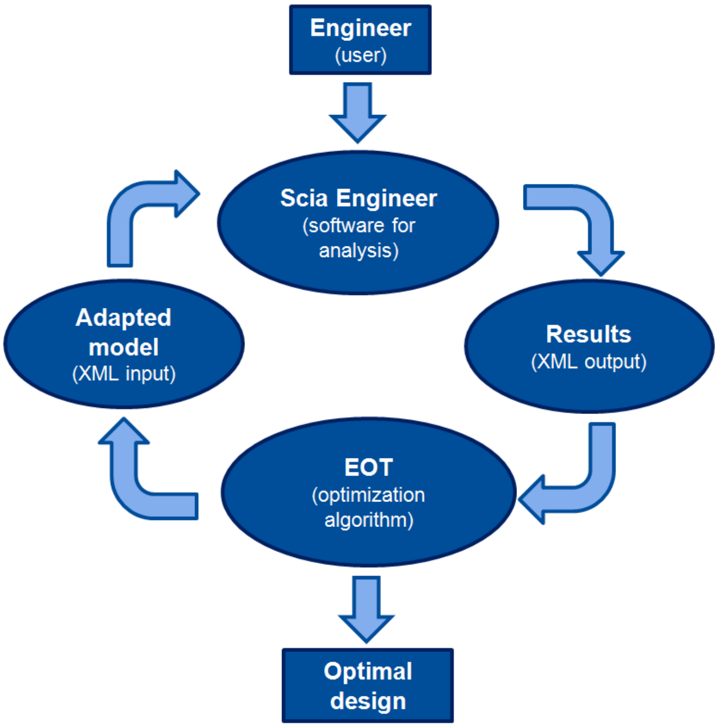



Fig. 5.1 Optimization process

### 5.1.1 Preparation of the model and its parameterization

The model of the analysed structure is created using standard Scia Engineer tools and functions. The geometry, boundary conditions, loads, etc. are defined. Parameters are assigned to the properties that can vary during the optimization. Parameters indicate that a particular property becomes variable and that the engineer defines its initial value and, if required, also the limits. If suitable or needed, it is possible to specify also relations between individual parameters by the formulas (e.g. the relation between the width and height of a cross-section).

Generally, the parameters can be of two kinds, input parameters and output ones. Input parameters can control properties of selected objects. Output parameters can be used for handling output values calculated by Scia Engineer (e.g. value of ratio of acting force to resistance, minimal deflection, member or structure weight etc.). By means of formulas, the user can define the objective function as a price or total structure weight etc.

### 5.1.2 Definition of the objective function and constraints

The objective function defines what is to be optimized. It can be the cost, mass, deflection and others. Currently, one-criterion optimization with constraints has been implemented. In addition to objective function, the sets of constraint are usually defined.

These can be ratios of acting force to resistance, maximal deformation, maximal stresses etc.

### 5.1.3 Selection of the optimization method

Furthermore, it is necessary to select one of the available optimization methods. As already stated in the previous chapters, the four basic optimization algorithms have been developed (DE, MSA, SQP and NM). The important fact is that each method is not suitable for optimization of general problems. The selection of the method may also affect the time needed for the solution of the sought-after result.

### 5.1.4 Optimization cycle

The optimization solver (EOT) generates the sets of parameters used for the creation of particular variants of the model. Scia Engineer receives these parameters, runs the prescribed calculations and code-check. In the next step, EOT gets back the results and evaluates them to modify the parameters in order to get closer to the desired optimal solution. This process is then repeated until the optimum is found. The communication between the optimization module (EOT) and Scia Engineer is based on XML format document.

### 5.1.5 Evaluation of the optimal solution

As already stated, the optimization finds one or more optima. It is the engineer who compares them and makes the final decisions. Results from each iteration are stored in EOT and can be examined after finalization of the optimization procedure by post-processing tools.

# 6 POST-TENSIONED SIMPLY SUPPORTED MEMBER (SPECIMEN)

The usability of the optimization methods is verified on the specimen at first. The simply supported 10 metres long member with rectangular cross-section is made from concrete class C30/37 (see Fig. 6.1). The permanent uniform load g = –30 kN/m is applied on the beam. The minimal distance between axis of prestressing tendon and edge of the cross-section is set as $c$ = 50 mm. The strands from material Y1770S7-12.5 are used for tensioning of the beam with initial stress 1440 MPa. Generally the prestressing losses are not considered in this example for simplification. The results from optimization process are compared with predesigned values based on the allowable concrete stresses.

The concrete stresses from linear combination (prestress + permanent) are evaluated. The maximal allowable values of concrete stresses are taken into account according to Chapters 5.10.2.2(5) and 7.2(2) from EN1992-1-1 for compressive stress $\left(\sigma_{\mathrm{cc,lim}} = 0.6 \cdot f_{\mathrm{ck}} = 0.6 \cdot 30 = -18\,\mathrm{MPa}\right)$. Tensile stress in concrete is not allowed at all ($\sigma_{\mathrm{ct,lim}} = 0\mathrm{MPa}$).

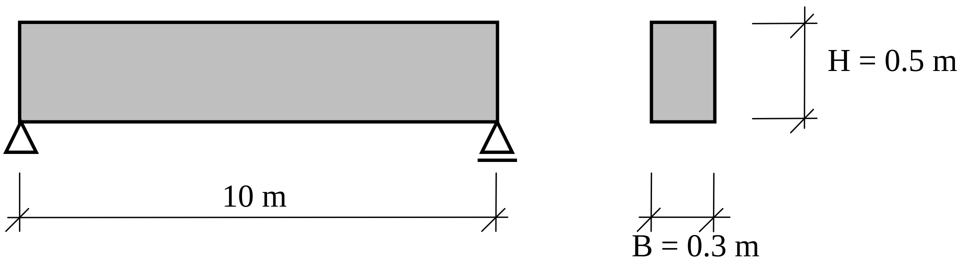



Fig. 6.1 Structural scheme of specimen

The target of this example is to verify the usability of optimization algorithm for design of prestressed concrete structures. The geometry of prestressing tendon is modified to get minimal amount of prestressing reinforcement in section within fulfilling of constraints. The objective function is expressed as the following

$$f(x) = \min\left(A_{\mathrm{p}}\right) = A_{\mathrm{p1}} \cdot n_{\mathrm{t}} \cdot n_{\mathrm{g}}. \tag{6.1}$$

where $A_{\mathrm{p1}}$ is an area of prestressing strand (Y1770S7-12.5A; $A_{\mathrm{p1}}$ =93 mm$^2$); $n_{\mathrm{t}}$ is a number of strand in tendon; $n_{\mathrm{g}}$ is a number of the tendon in group.

Two types of tendon geometry are used for the verification of proposed optimization algorithms.

- Type 1 – geometry consisting of two parabolic arcs; see Fig. 6.2
  - Parabola + tangent (end)
  - Parabola + tangent (begin)

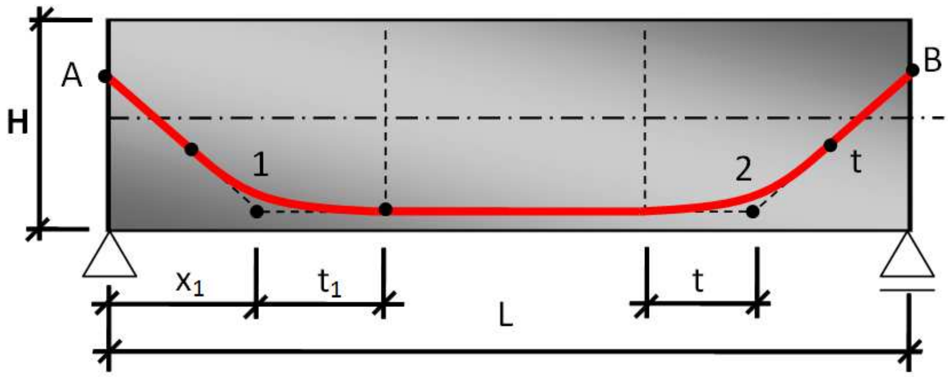



Fig. 6.2 Tendon geometry – type 1

- Type 2 – geometry consisting of one symmetrical parabolic arc; see Fig. 6.3
  - Symmetrical parabola + tangent

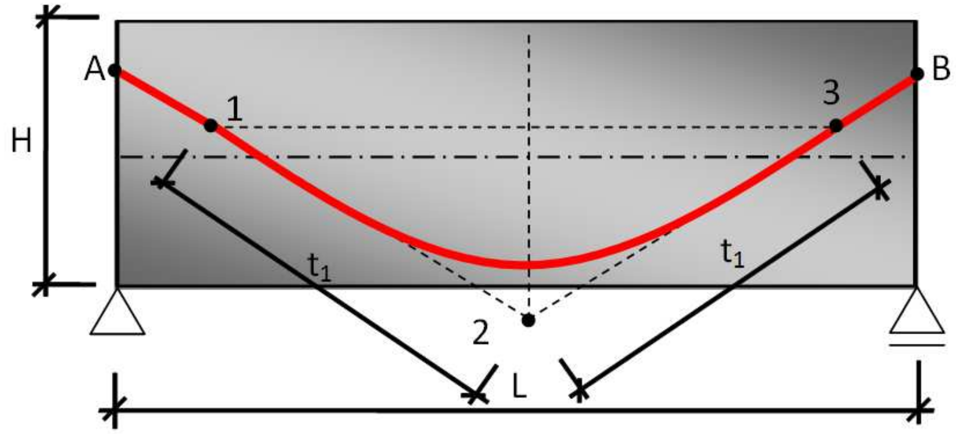



Fig. 6.3 Tendon geometry – type 2

A predesign of the prestressing reinforcement was performed according to allowable concrete stresses. The geometry of tendon without eccentricities at the ends ($e_{pA} = e_{pB} = 0$ m) and maximal possible in the centre of the beam $e_{p2} = -0.2$ m (for type 2 $e_{pv} = -0.2$ m) were considered as the optimum tendon geometry. Short straight half metre parts of tendon were selected at the both ends. The minimal prestress force can be obtained from the following formulas based on the stresses in bottom (1) and top (2) fibres of cross-section.

$$P_{1,c} \geq \frac{-(W_1 \cdot \sigma_{cc,lim} - M_g)}{e_p + j_2} = \frac{-(0.0125 \cdot (-18 \cdot 10^3) - 375)}{0.2 + 0.083} = 1323.53\text{kN}, \tag{6.2}$$

$$P_{1,t} \leq \frac{-(W_1 \cdot \sigma_{ct,lim} - M_g)}{e_p + j_2} = \frac{-(0.0125 \cdot 0 - 375)}{0.2 + 0.083} = 2117.65\text{kN}, \tag{6.3}$$

$$P_{2,c} \leq \frac{(W_2 \cdot \sigma_{cc,lim} - M_g)}{e_p - j_1} = \frac{(0.0125 \cdot (-18 \cdot 10^3) - 375)}{0.2 - 0.083} = 3214.29\text{kN}, \tag{6.4}$$

$$P_{2,t} \geq \frac{(W_2 \cdot \sigma_{ct,lim} - M_g)}{e_p - j_1} = \frac{(0.0125 \cdot 0 - 375)}{0.2 - 0.083} = 3214.29\text{kN}, \tag{6.5}$$

where $W_{1,2}$ are the section modulus, $M_g$ is a moment from external permanent load and $j_{1,2} = \frac{W_{1,2}}{A_c}$. These formulas result to predesigned optimal value of prestressing force ($P_d = 1323.53\text{kN}$) which fits all limits above.

$$A_{p,d} = P/\sigma_p = 1323{,}53/(1440 \cdot 1000) = 919\ \text{mm}^2, \quad (6.6)$$

$$n_{p,d} = A_{p,d}/A_{p1} = 919/93 = 9{,}88 \approx 10\ \text{strands}. \quad (6.7)$$

The alternatives 2 pieces of 5 strands tendon or 1 piece of 10 strand tendon are estimated according to previous formulas. The tendon geometries along the whole beam are the same for both type of geometries in case of the design variables mentioned in Tab. 6.1.

Tab. 6.1 Values for predesigned optimal solution

| Parameter | Type 1 | Type 2 | Explanation |
|---|---|---|---|
| $e_{pA}$ [m] | 0.0 | 0.0 | eccentricity of prestressing in point A |
| $e_{p2}$ [m] | –0.2 | –0.36 | eccentricity of prestressing in point 2 |
| $x_1$ [m] | 2.75 | - | distance of cross-link of tangents for the beginning of the beam |
| $t_1$ [m] | $x_1 - 0.5 = 2.25$ | 4.5 | length of tangent |

## 6.1 Constraints

Generally constraints are needed almost for each optimization task. This example is also constrained optimization example. The constraints can be set for design variables (minimal and maximal values) and for the evaluated parameters. The following constraints were used.

- Geometrical – this constraint is valid for geometry type 2 only, the top point of symmetrical parabola has be inside of cross-section and it has to satisfy the concrete cover

$$g_1(x) = -\frac{e_{pV}}{\frac{H}{2} - c} - 1 \le 0. \quad (6.8)$$

- Allowable concrete stresses
  - In tension – no tension allowing can be expressed as follows

$$g_2(x) = \sigma_{ct} \le 0\ . \quad (6.9)$$

- In compression – maximal stress in compression is limited by 60% of characteristic cylinder strength of concrete $\left(\sigma_{\mathrm{cc,lim}} = 0.6 \cdot f_{\mathrm{ck}}\right)$ can be written in term

$$g_3(x) = \frac{\sigma_{cc}}{\sigma_{\mathrm{cc,lim}}} - 1 \leq 0\ . \qquad (6.10)$$

## 6.2 Used optimization algorithms

Four different optimization techniques are used for the finding optimal shape of tendon geometry. Those methods are

- sequential quadratic programming (SQP),
- Nelder-Mead (NM),
- modified simulated annealing (MSA),
- differential evolution (DE).

As you can see further, the initial values of design variables are intentionally selected more different than values for predesigned solution.

### 6.2.1 Sequential quadratic programming (SQP)

As was described in Chapter 4.2.1, the method is suitable only for design using continuous variables. A necessary area of prestressing tendon is based on the number of strands in tendon and number of the same tendon. Both values are discrete variables. Therefore SQP method is not usable for this optimization task in case of modelling real prestressing tendon in the structure. Replace real tendon by the equivalent load is another possibility of modelling. Prestressing tendon was replaced by the polygon for graphical drawing only. Parameterization of equivalent load has to cover all changes of tendon geometry. Tendon geometries are displayed in Fig. 6.5 and Fig. 6.7.

### 6.2.2 Nelder-Mead method

Similarly as for SQP method, also NM method enables optimization only with continuous variables. Thus, the post-tensioned tendon needs to be substituted by the equivalent load representing the prestressing.

### 6.2.3 Modified simulated annealing (MSA)

MSA method is very robust algorithm with small error sensitivity. The big amount of iterations and relative not sufficient accuracy of optimal solution can be named as disadvantages. Nevertheless, the method is successfully used for optimization task with discrete variables (number of strands in tendons in our case). Therefore, the method is applied in case of real defined prestressing tendon. The parameterization of number of strands and number of tendon is shown in Fig. 6.4. The prestressing was modelled using post-tensioned tendon with two parabolic arcs (see Fig. 6.5 and Fig. 6.7).

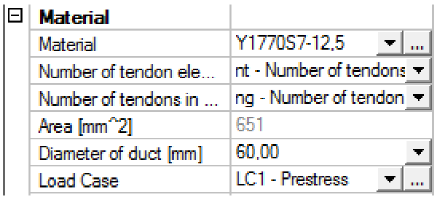



Fig. 6.4 Parameters of tendon properties

### 6.2.4 Differential evolution (DE)

Differential evolution can be also used together with discrete variables (with defined real tendon). The searching was performed from 20 member's population with maximal 100 numbers of iterations.

## 6.3 Type 1 (two parabolic arcs)

Tendon geometry consists from two parabolic arcs – parabola + tangent (end); parabola + tangent (begin). All four optimization techniques were used. The settings for initial and limit values for all methods of case type 1 is mentioned in Tab. 6.2. The different initial values are used for number of strands and number of the same tendon in the group ($n_t$; $n_g$).

Tab. 6.2 Initial and limit values for optimization method of type 1

| Method | SQP | | | NM; MSA; DE | | |
|---|---|---|---|---|---|---|
| Parameter | Initial | Minimum | Maximum | Initial | Minimum | Maximum |
| $n_t$ [-] | 7 | 6 | 19 | 12 | 6 | 19 |
| $n_g$ [-] | 2 | 1 | 10 | 3 | 1 | 10 |
| $e_{pA}$ [m] | 0.1 | 0 | 0.1 | 0.1 | 0 | 0.1 |
| $e_{p2}$ [m] | –0.1 | –H/2 + c | 0 | –0.1 | –H/2 + c | 0 |
| $x_1$ [m] | 1.8 | 1.5 | 2.75 | 1.8 | 1.5 | 2.75 |

Tendon is composed from two parabolic arcs which are symmetrically placed to the middle of the beam. The parabolic arc with vertex point $x_1$ is defined using tangent length $t_1$. Tangent length is dependent on distance to vertex point $t_1 = x_1 - 0.5$ m.

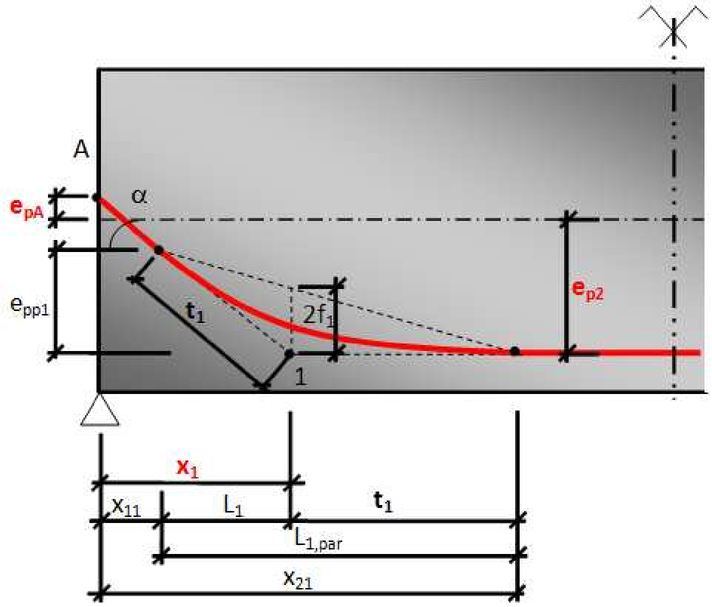



Fig. 6.5 Type 1 (red are independent variables)

The comparison of the results for all used optimized algorithm is performed in Tab. 6.3. The necessary amount of prestressing reinforcement was predesigned using concrete stresses, the optimization procedure found amount of prestressing reinforcement in correspondence of constraints satisfying. When the obtained values of required area of prestressing reinforcement ($A_{p,req}$) are rounded to integers we obtained 1 piece of 11strands tendon for SQP method. Method MSA converged to group of optimal solution after 3278 iterations. The best member from population is 1 piece of 10 strand tendon similarly like for NM method. Differential evolution finished with 1 piece of 11 strand tendon. The optimized tendon geometries are slightly different from predesigned optimal geometry. Nevertheless, they can be used. The graphical comparison of optimized tendon geometry together with predesigned initial one is displayed in Fig. 6.6.

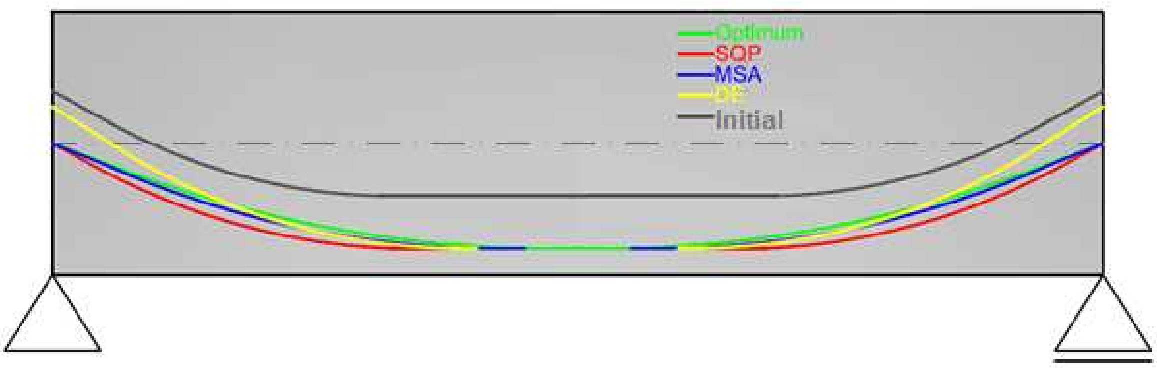



Fig. 6.6 Graphical comparison of the predesigned and optimal tendon geometry

Tab. 6.3 Comparison of predesigned and optimal values (type 1)

| Parameter | Initial | Predesigned | Optimized | | | |
|---|---|---|---|---|---|---|
| | | | SQP | NM | MSA | DE |
| $n_t$ [-] | 7 | 10 | 8.50→11 | 10.35→10 | 10 | 9.74→11 |
| $n_g$ [-] | 2 | 1 | 1.19→1 | 1 | 1 | 1.05→1 |
| $e_{pA}$ [m] | 0.1 | 0.0 | 0.0 | 0.037 | 0 | 0.07 |
| $e_{p2}$ [m] | –0.1 | –0.2 | –0.2 | –0.199 | –0.2 | –0.19 |
| $x_1$ [m] | 1.8 | 2.75 | 1.85 | 2.29 | 2.5 | 2.15 |
| $A_{p,req}$[mm$^2$] | 1302 | 930 | 941→1023 | 930 | 930 | 951→1023 |
| max $\sigma_x$ [MPa] | 3.02 | –9.12 | –9.71 | –6.0 | –8.90 | –13.59 |
| min $\sigma_x$ [MPa] | –33.22 | –17.56 | –16.17 | –15.62 | –17.50 | –17.94 |
| No. of iterations [-] | - | - | 105 | 175 | 3278 | 540 |
| Total opt. time [min] | - | - | 7:36 | 25:10 | 3:56:24 | 38:47 |

## 6.4 Type 2 (symmetrical parabola)

The second tendon geometry consists of symmetrical parabola with vertex point in the middle of the beam ($x_2 = L/2$). Parabolic arc is defined using tangent length $t$. It was necessary to define several additional parameters and formulas to cover all changes in geometry.

Tab. 6.4 Initial and limit values for optimization method of type 1

| Method | SQP | | | MSA; NM; DE | | |
|---|---|---|---|---|---|---|
| Parameter | Initial | Minimum | Maximum | Initial | Minimum | Maximum |
| $n_t$ [-] | 7 | 6 | 19 | 12 | 6 | 19 |
| $n_g$ [-] | 2 | 1 | 10 | 3 | 1 | 10 |
| $e_{pA}$ [m] | 0.1 | 0 | 0.1 | 0.1 | 0 | 0.1 |
| $e_{p2}$ [m] | –0.2 | –H | 0 | –0.2 | –H | 0 |
| $t_1$ [m] | 1.5 | 1.0 | 4.5 | 1.5 | 1.0 | 4.5 |

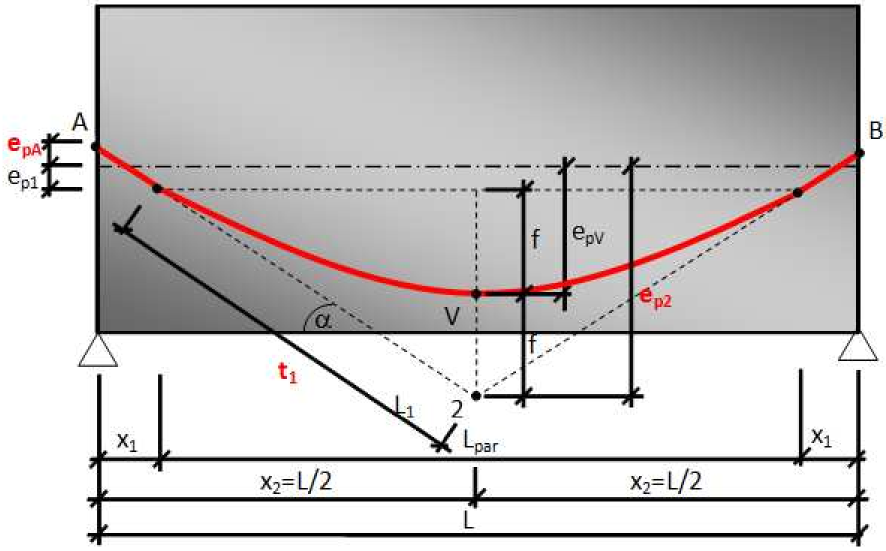



Fig. 6.7 Type 2 (red are independent variables)

The comparison of the optimized results with initial predesigned values is performed in Tab. 6.5. When initial values ($n_t$ and $n_g$) are selected according to Tab. 6.2, then SQP method converged to optimum but did not find better solution than predesigned. Therefore initial values ($n_t = 12$ and $n_g = 3$) were used the same like for genetic algorithm. Unfortunately, the optimization process did not find the similar necessary area of prestressing reinforcement as predesigned 10 strands tendon.

Tab. 6.5 Comparison of predesigned and optimal values (type 2)

| Parameter | Initial | Predesigned | Optimized | | | |
|---|---|---|---|---|---|---|
| | | | SQP | NM | MSA | DE |
| $n_t$ [-] | 7 | 10 | 11.48→12 | 10.42→10 | 10 | 10 |
| $n_g$ [-] | 2 | 1 | 1.00→1 | 1 | 1 | 1 |
| $e_{pA}$ [m] | 0.1 | 0.0 | 0.0 | 0.02 | 0.04 | 0.005 |
| $e_{p2}$ [m] | –0.1 | –0.36 | –0.24 | –0.27 | –0.325 | –0.285 |
| $t_1$ [m] | 1.8 | 4.5 | 1.65 | 3.2 | 3.5 | 3.0 |
| $A_{p,req}$[mm$^2$] | 1302 | 930 | 1116 | 930 | 930 | 930 |
| $e_{pV}$ [m] | –0.155 | –0.198 | –0.200 | –0.195 | –0.197 | –0.198 |
| max $\sigma_x$ [MPa] | 3.02 | –12.73 | –10.70 | –12.65 | –13.17 | –9.45 |
| min $\sigma_x$ [MPa] | –33.22 | –17.35 | –17.57 | –17.23 | –17.76 | –17.70 |
| No. of iterations [-] | - | - | 75 | 543 | 3168 | 2020 |
| Total opt. time [min] | - | - | 5:17 | 38:15 | 3:48:43 | 2:25:35 |

Modified simulated annealing method converged to optimum after 3168 steps of iterations. The best solution is the identical for the second tendon geometry (type 2) like for the first one (type 1). The optimum is 1 piece of 10 strand tendon. The same results for both types of geometry are obtained for method of differential evolution (1 piece of 10 strand tendon).

Finally, the same necessary area of prestressing reinforcement was found using optimization algorithms. When we compare the optimal and predesigned tendon geometries the very good approximation was obtained as shows Fig. 6.8. The optimized eccentricity of tendon in the middle of the beam ($e_{pV}$ = –198 mm) is very close to maximum ($e_{pV,max}$ = –200 mm). The depths are 5 times scaled.

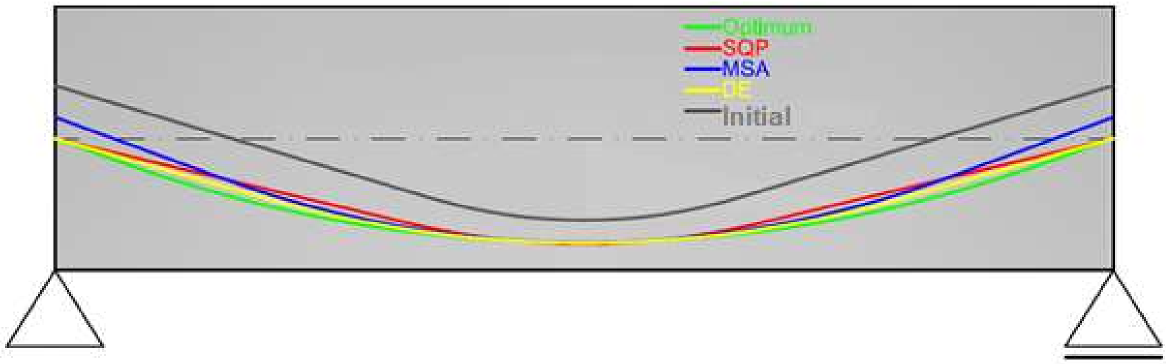



Fig. 6.8 Graphical comparison of the predesigned and optimal tendon geometry (type 2)

## 6.5 Sensitivity analysis

Previously, an optimal design of post-tensioned concrete simply supported beam using optimization algorithms was demonstrated for two basic tendons geometries. Now sensitivity analysis is applied on this example. In any optimization task a small change in the design variables can affect the optimal solution. Sensitivity analysis is reported for design problem of tendon geometry type 1. The eccentricity of the tendon is taken using maximum practical value in the middle of the beam with respect to concrete cover and annual eccentricity at the both ends. The assumption seems to be more realistic than determine optimal tendon geometry for this simply supported member. The minimization of total cost and total weight is analyzed with respect of effect (a) beam span, (b) beam height and (c) concrete characteristic compressive strength.

An effect of input values on optimal design variables is studied in this part. The optimal values of cross-section dimensions and prestressing forces are looking for typical span lengths (from 10 to 24 m with step 2.0 m) for investigation of beam span influence.

The impact of the cross-section width for the optimal design of beam is performed for several typical cross-section widths (from 0.2 m to 0.5 m with step 0.05 m). The last criterion of sensitivity analysis is the characteristic cylinder concrete strength. Five different concrete classes have been studied with different prices per cubic metre (see Tab. 6.6). The sensitivity analysis is performed on three typical cross-sections (rectangular, T-section and I-section) with the following dimensions (see Fig. 6.9). The dimensions of flanges for T and I sections are considered dependently of the depth and width of the cross-sections (sB = 2.25 × *B*; sH = *H*/4).

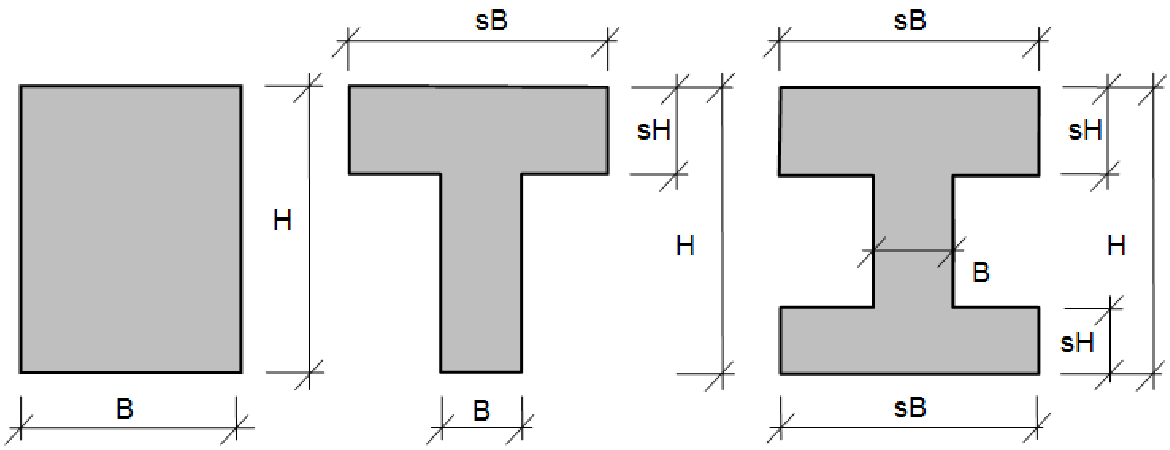



Fig. 6.9 Typical cross-section used for sensitivity analysis

### 6.5.1 Objective function

An objective function in previous chapters was focused on amount of prestressing reinforcement only. Nevertheless, the dimensions of cross-section were fixed for optimization. During this sensitivity analysis the objective function is defined as total cost for concrete and prestressing reinforcement together. Volume of concrete ($V_c$) and total weight of prestressing tendons ($M_p$) is evaluated. Finally costs for concrete ($C_c$), prestressing tendon ($C_p$) and total cost ($C_m$) are calculated according to formulas

$$C_c = C_{unit,c} V_c \,, \tag{6.11}$$

$$C_p = C_{unit,p} \; M_p \,, \tag{6.12}$$

$$C_m = C_c + C_p \,. \tag{6.13}$$

Simultaneously, the dimensions of cross-section are optimized for each cross-section type, length of the beam, width of the beam and concrete class. The prices per cubic metre and kg were considered in this study as mentioned in Tab. 6.6. The exchange rate is fixed in the whole dissertation to 25.0 CZK per 1 Euro.

Tab. 6.6 Cost of different concrete classes

| Concrete class | $C_{\text{unit,c}}$ [CZK / m$^3$] | $C_{\text{unit,c}}$ [€ / m$^3$] |
|---|---|---|
| C 16/20 | 4 650.0 | 186.0 |
| C 20/25 | 4 750.0 | 190.0 |
| C 25/30 | 5 000.0 | 200.0 |
| C 30/37 | 5 200.0 | 208.0 |
| C 35/45 | 5 400.0 | 216.0 |

Let us assume typical prices including formwork and labour for concrete casting and reinforcement placement. The cost for a kilogram of 15.7 mm strand is taken into account as $\mathrm{C}_{\text{unit,p}}$ = 4.0 €/kg. The comparison criteria are length of the beam, depth of the cross-section and compressive cylinder strength of concrete.

### 6.5.2 Constraints

Constraints are used the same like for specimen type geometry 1. The beams are optimized on allowable concrete stresses.

- In tension – no tension is allowed expressed as

$$g_1(x) = \sigma_{\text{ct}} \leq 0. \tag{6.14}$$

- In compression – maximal stress in compression is limited by 60% of characteristic cylinder strength of concrete $\left(\sigma_{\text{cc,ch}} = 0.6 \cdot f_{\text{ck}}\right)$ written as

$$g_2(x) = \frac{\sigma_{\text{cc}}}{\sigma_{\text{cc,ch}}} - 1 \leq 0. \tag{6.15}$$

The sensitivity analysis covers also optimization of cross-section dimensions. Therefore, the geometrical constraints were set for optimization process. The ratio between depth and width of the all cross-sections was set to limits.

- Minimum ratio – depth of cross-section has to be higher than width

$$g_3(x) = \frac{H}{B} - 1 \geq 0. \tag{6.16}$$

- Maximum ratio - depth of cross-section has to be maximally two times higher than width

$$g_4(x) = \frac{H}{2B} - 1 \leq 0. \tag{6.17}$$

### 6.5.3 Results

The optimization algorithm has been run for each comparison parameter. Generally, one optimization problem was analyzed for each cross-section type, length of the beam, width of the cross-section and concrete class. Finally, there were about 60 different optimization examples. Considering number of strand in tendon as discrete variable the optimization algorithm was chosen enabling handle with discrete parameters. Therefore, modified simulated annealing method was used for the optimization of this sensitivity analysis. A population of 20 members was analyzed in 10 simulated annealing steps. The obtained results from modified simulated annealing were improved using sequential quadratic programming method. This is the combination of two algorithms. In the first step robust algorithm (MSA) searches roughly good solution and the second step applies accurate algorithm (SQP) improving the existing solution. Each iteration was finished after 15 constraints satisfied solution or 150 running solution.

The effect of changing the beam length (*L*) is shown in Fig. 6.10-Fig. 6.11. As expected the width of the beam increases with the increasing of the length for all cross-sections. T and I-sections are very parallel. The increase of cross-section width is significant for rectangular section mainly.

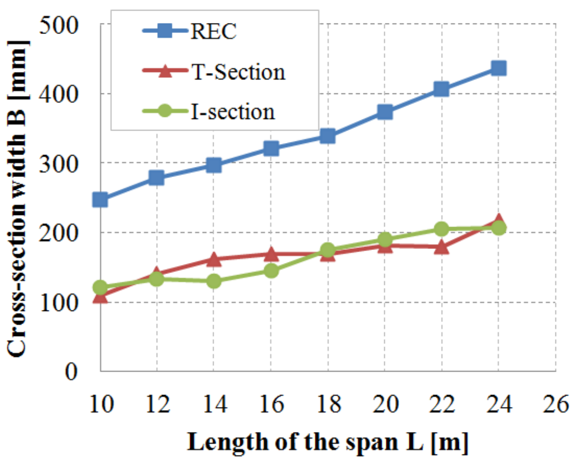



Fig. 6.10 Span effect on beam width for C30/37

The similar behaviour is related to prestressing forces. The prestressing forces are the same for T and I-sections to length 12 m. From 14-22m is I–section close to rectangular. Finally, the optimal width returned back near to T-section for 24 m.

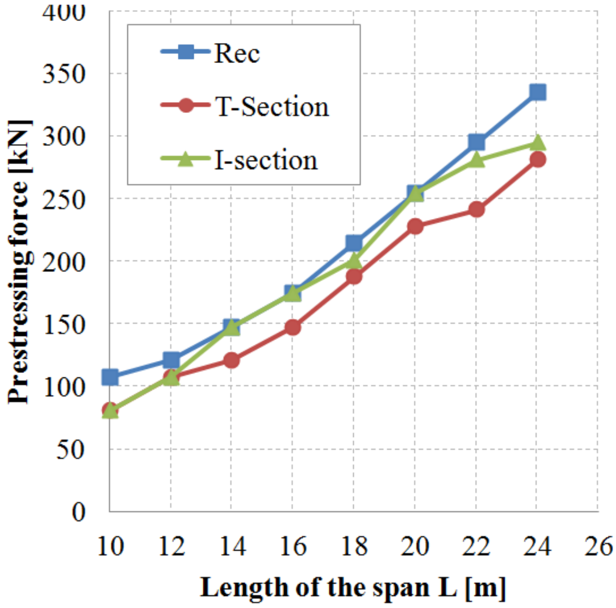



Fig. 6.11 Span effect on the prestressing force for C30/37

A total cost increases with the growing length for all cross-sections (see Fig. 6.12). T-section shows the less total cost. The most expensive are the rectangular cross-section for all span lengths.

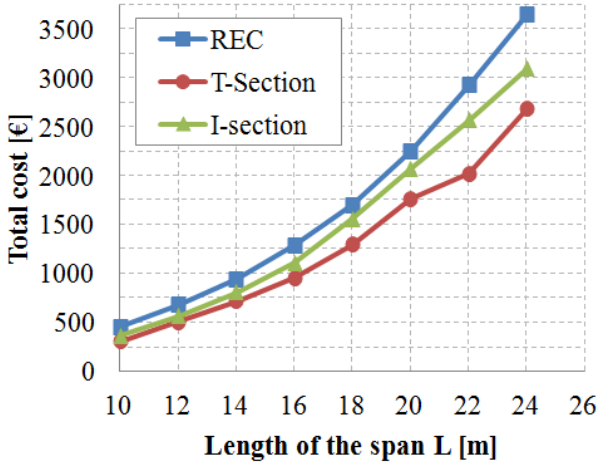



Fig. 6.12 Span effect on total cost for C30/37

The last investigated parameter was characteristic cylinder strength of concrete. The five different concrete grades were used (C16/20, C20/25, C25/30, C30/37, C35/45) on beam with $L$ = 10 m. Each concrete has different strength parameters. Therefore, the maximal compressive strength of concrete is dependent on the concrete class. The prices per cubic metre of concrete were considered according to Tab. 6.6. The results based on the total weight of structure are compared in Fig. 6.13. As expected the significant decreasing of structural weight depends on increasing of concrete class. The difference between concrete C16/20 and C35/45 is more than 1200 kg.

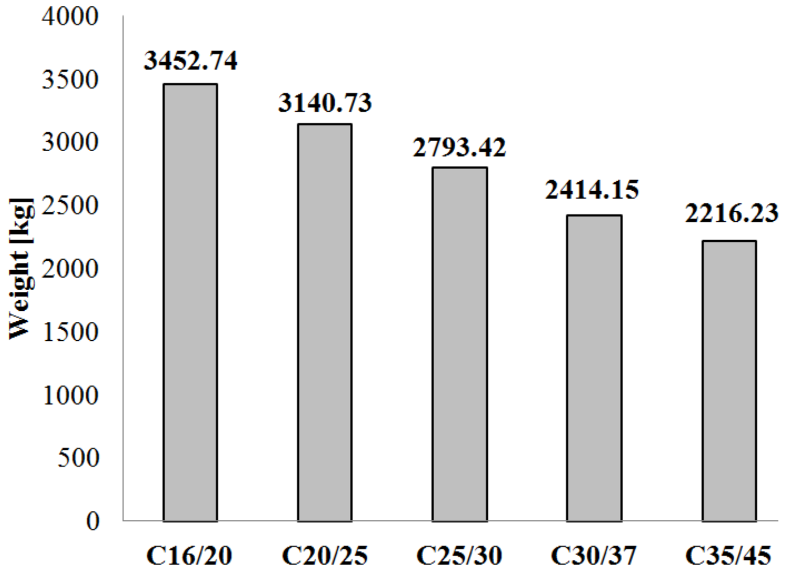



Fig. 6.13 Comparison of total weight of the beam

While comparing total cost (see Fig. 6.14) instead of total weight of beam the tendency for concrete classes is the same but the differences between the results for each class is not such significant. The results for all concrete grades respect the range of only 20 €. As a conclusion of this test, the comparison of total price seems to be more objective than comparison of the total weight of the structure.

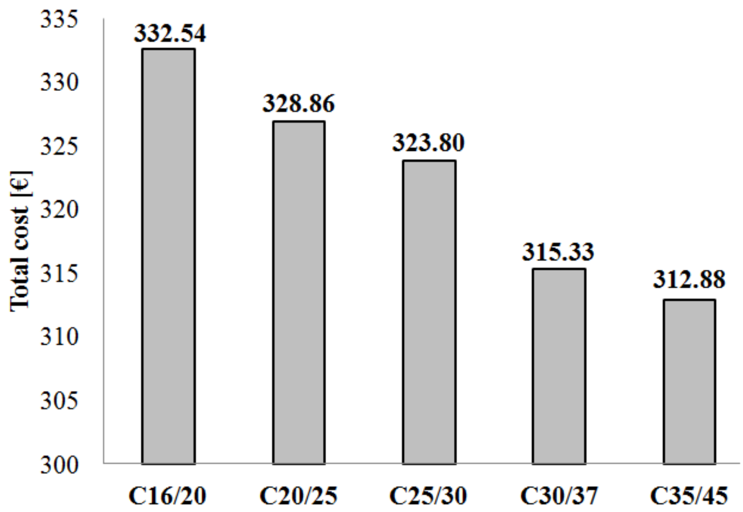



Fig. 6.14 Comparison of total cost of the beam

## 6.6 Conclusions

A verification of proposed method for simple prestressed concrete structure has been target of this Chapter. However, the proposed procedure from Chapter 5 is applicable to any kind of the structure as we focus on the design of prestressed structures. The testing resulted, which optimization algorithms are the most suitable for optimization of prestressed concrete structures with their advantages and disadvantages.

Consequently, the SQP method provides very good solutions in perfect time in comparison with other algorithms. One of the main disadvantages is a very limited use on a real number of tendons defined as integer. This method can be used when the effects of prestressing are replaced by the equivalent load. Nevertheless, a substitution of the tendon decreases comfort of work. Additionally, the short-term and long-term losses can be estimated only. It can be pointed out that SQP is not a robust method (see 2.1.5). Therefore, the selection of initial parameters is an important step during the optimization process. When the initial values are chosen, the algorithm reaches to the local extreme and it is not possible to get out of it. Similarly like SQP, the NM method also found acceptable solution. Nevertheless, the same disadvantages like for SQP cause that this method not suitable for the optimization of the number of tendons.

The application of the MSA method is advantageous in many respects. This algorithm found better solutions than SQP and the choice of initial parameters is not significantly important. However, the required time for optimization is the main disadvantage. The use of the method can be limited especially for the optimization of large structures with construction stages analysis. Hence, it is necessary to set the size of population, number of iteration and step of design variables with respect to the maximal required optimization cycles. The acceptable time frames are considered minutes, hours and a day at most. The fourth used method (DE) provides a compromise between previous two. The optimized solutions are not that good but acceptable. The benefit of this method is relatively fast convergence and the use of integer values of tendons.

Our verification of simple example, which belongs to main conclusion of this Chapter, proved the most suitable methods for optimization of design variables of prestressed concrete structures are evolutionary algorithms as MSA and DE. Therefore, these methods will be used for the next examples. The use of these methods is very promising for the optimization of more complicated prestressed concrete structures.

# 7 OPTIMIZATION OF DESIGN VARIABLES IN POST-TENSIONED BRIDGES

Generally, the technology of post-tensioned concrete is commonly used in construction of medium and long span concrete bridges all around the world. An opportunity of the active modification of the internal forces distribution offered by a variable geometry of post-tensioned tendons is one of the main advantages. Unfortunately these kinds of structures are not usually investigated as optimization tasks. This is caused by a complexity of a structure and a presence of many design variables which are often discrete.

Several studies on the optimization of the post-tensioned concrete structures have been already published in the past. Typically, Marks and Trochymiak [83] presented a work dealing with the optimized design of a tendon geometry in continuous concrete three spans bridge box girders using linear programming. The objective function was focused only on prestressing reinforcement. Design constraints were considered as normal concrete stresses and tendon eccentricities with respect to concrete cover according to Polish code. Similarly, Quiroga and Arroyo [99] published a study with an optimized geometry of prestressing tendons on fixed cross-sectional dimensions. Again, normal stresses of concrete served as constraints. Both these contributions used optimization techniques based on mathematical programming. One of the first examples employing structural requirements leading to discrete formulation of the design problem, Martí and González-Vidosa [84] published a study focused on an optimal design of a prestressed foot bridge using a Simulated Annealing method. The objective function consists of a cost for concrete, nonprestressed and prestressing reinforcement. Constraints were ultimate bending moment capacity, shear capacity and a deflection check according to Spanish EHE code.

However, the mentioned models were solved using linear analysis and did not take into account real behaviour of a structure with construction stages and time effects together with optimization procedure. Nowadays European standards (Eurocodes) are widely used almost in the whole of Europe, therefore our optimization is focused on the design and checks according to the design code for concrete bridges (EN1992-2). To the best authors' knowledge, such kind of structural optimization has not been presented in available literature yet. This chapter presents the proposals, recommendations and several typical examples of post-tensioned concrete bridge optimization. We focus on the decreasing of required numbers of strands in the tendon or completely neglect particular tendon in the structure. Simultaneously, the optimization methods find such tendon geometry (X, Z nodes coordinates of tendon geometry) to get maximal efficiency of the concrete checks in the structure defined as ratio of acting internal forces divided by cross-section resistance.

## 7.1 Optimization process -definition of optimization task

When we optimize post-tensioned structure it is necessary to establish several preparatory steps. A fulfilment of them produces a successful progress and achievement of the optimal solutions. The optimization process can be divided into the following steps:

- general overview of the structure
- parameterization of structure
- definition of objective function
- selection of the mathematical algorithm for optimization
- post-processing analysis and verification of the results

### 7.1.1 General overview of the structure

In the beginning of the each optimization task, it requires producing of general overview of the structure. All information about the optimized structures are collected and analyzed. The structural model, material properties, required road category, and acting loads of the structure are the essential requirements for the start. The fundamental dimensions of the structural members are estimated using engineer experience and knowledge. Based on these proportions, the first rough solution is introduced. Usually, the initial solution does not have satisfied the particular constraints. Generally, a varying distribution of the tendons provides the possibility of active distribution of the internal forces from the prestressing in the structure. Therefore, the optimization tasks focus on searching the most suitable tendon geometry to get minimal tendon mass and maximal structural efficiency.

A typical case can be distribution of the tendon geometries in the construction joint of the bridge with cantilever overhang like mentioned in Fig. 7.1. Here, the small change of the vertical coordinates significantly influence the internal forces in the both construction parts. Generally, three different tendons are optimized by the change of the vertical positions in the joint. The first tendon stresses over the original span with overhang. Furthermore, the couplers connect the tendons in the construction joint and prestress the built span. The last is introduced due to impossible coupling of all tendons in the construction joint. These tendons are continuous over two adjacent spans. An application of the optimization methods on this case describes in Chapters 7.3 and 7.4. In case of more detailed optimization, it is also possible to deal with radius of tendon arcs.

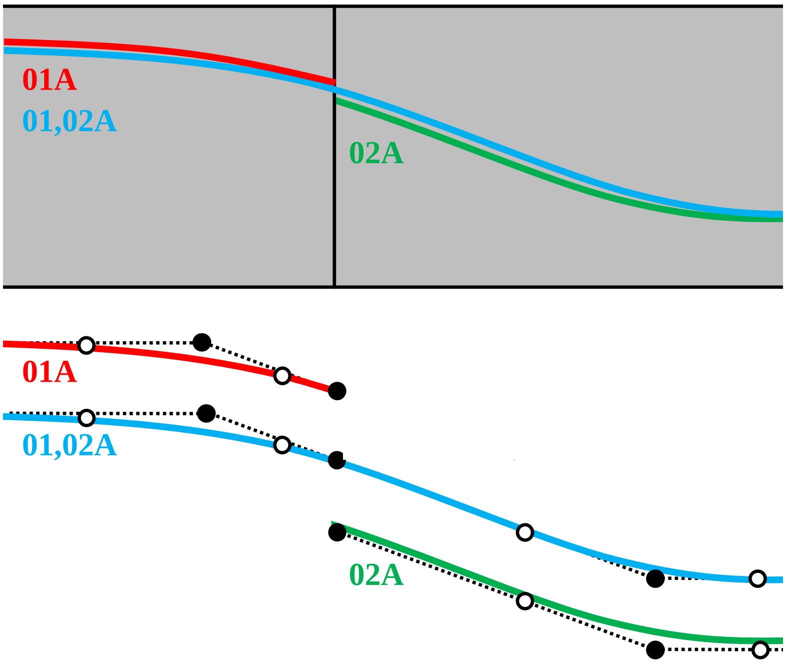



Fig. 7.1 Tendon distribution in the construction joint

### 7.1.2 Parameterization of the structure

As was mentioned in the Chapter 2.1.7, a parameterization itself is the most important step during the optimization process. Many parameters usually cause huge number of iterations. However, several parameters only with large dependency with the others usually produce impossible achievement of the convergence criteria. The most suitable number of parameters requires experienced estimation and knowledge of some characteristic structures. In addition, the well understanding of structural behaviour gives the better overview. When we optimize prestressed structure three groups of the parameters can be independent variables:

- Tendon geometry
- Number of tendons
- Tendon stressing characteristics

The items mentioned above are explained in details in the following chapters.

#### 7.1.2.1 Parameterization of tendon geometry

Let us consider a typical three span deck bridge. Relatively thick deck cross-section is sensitive for any small change of vertical tendon geometry. Three different tendon geometries are predefined (see Fig. 7.2). It is not necessary to parameterize all tendons coordinates as an independent variable. The coordinates significantly affecting the internal forces from prestressing are parameterized only. In this case, different z-coordinates of tendon A and C above the internal support and in the spans together with beginning of the tendon arcs are set as an independent variable. The figure gives the overview about parameters.

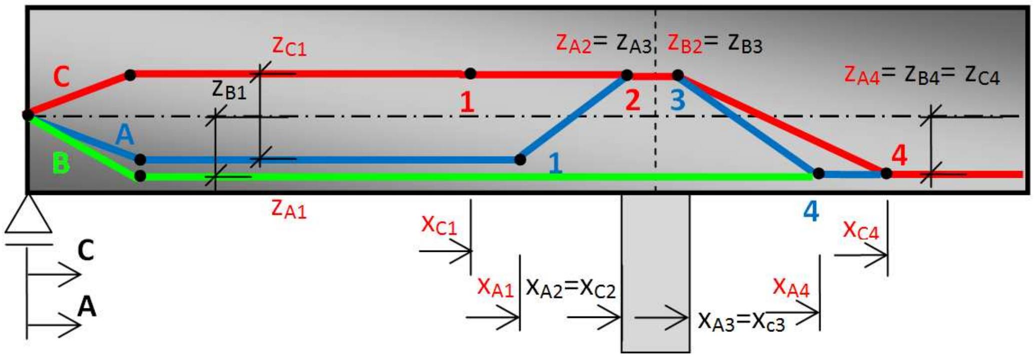



Fig. 7.2 Parameterization of the vertical geometry of tendons

#### 7.1.2.2 Parameterization of number of strands

Furthermore, the number of strands in the tendon and number of tendons are the most important variables influencing the objective function. In case of relatively simple structure in Fig. 7.2, all those numbers are parameterized (only 6 independent variables). The increasing of the parameters numbers is evident from the more complicated structure where more different tendon geometries appear. The typical case is the structure mentioned in the Chapter 7.3. Double T-section post-tensioned bridge includes 10 different geometries in one girder. Totally 20 variables for number of strands in tendon and 20 variables for possible tendon with the same geometry can be defined. From the practical reason, the tendons with the same characteristics and geometry are considered as one design object. Finally, this structure includes four different independent variables (two for number of strands in tendon and two for number of tendons).

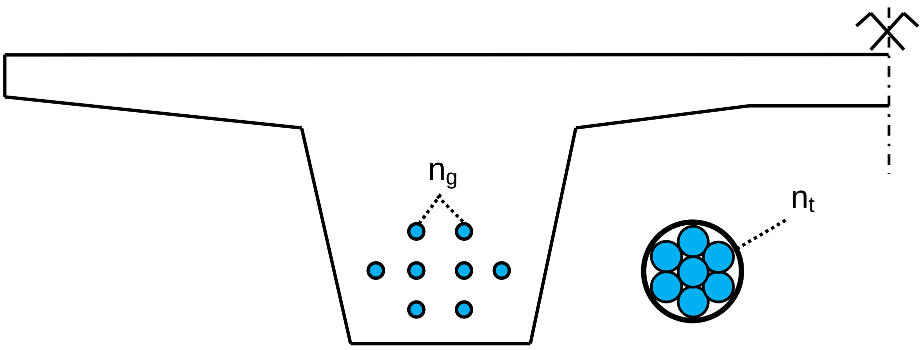



Fig. 7.3 Tendon arrangement in the two-girder bridge

#### 7.1.2.3 Parametrization of tendon characteristics

A prestressing system is characterised by several settings. These influence the evaluation of tendon losses along the tendon length. An initial tendon stress affects the internal forces from prestressing. Therefore, the initial stress is usually selected as an independent variable. There are also other settings determining prestressing units as a type of prestressing procedure, values of anchorage sets, directions and order of the stressing and anchorage. Generally, these properties certify particular prestressing producer and parameters are optimized very seldom. The parameterization of the initial stress gives the possibility to active change of distribution of the internal forces in the structure. Sometimes, it can be more efficient to decrease initial stress and modify the tendon geometry towards to increase numbers of strand with fixed geometry. For instance in case of Freyssinet hinge bridges, it must not be important to have maximal eccentricity of tendon above the internal supports. Nevertheless, a changing of initial stress is limited by the constraint of allowable stresses in prestressing reinforcement.

#### 7.1.2.4 Range of the variables

During the optimization, the mathematical methods select the values of independent variables from the predefined limits. The minimum and maximum possible values provide available range of optimized parameters. A typical case is distance of the centre of tendon from the cross-section surface during the vertical geometry optimization. The minimal cover of the prestressing can be ensured by the maximal or minimal value (top or bottom surface). The second possibility how to verify this condition is introducing of the boundary constraints (see chapter 7.1.4).

#### 7.1.2.5 Step for the variables

When the limits for each independent variable are defined, the interval can be divided according to type of the variable. Generally, two types of variable exist. A continuous variable represents the first group not requiring a step value. If the number of strands is optimized, it is necessary to set the step value. An integer can be used only for the number of strands. As has been mentioned before (see Chapter 4), an optimization algorithm depends on the selected step value.

### 7.1.3 Definition of objective function

In case of optimization of post-tensioned bridges, the different purpose expresses the objective function. Generally, the total costs and total weight evaluate the optimization of the whole structure. Nevertheless, we intentionally define different objective function for particular bridge to show optimization possibilities of such kind of structures. We focus on the optimization of the geometry of the post-tensioned tendons and concrete cross-section dimensions.

1) Area of prestressing

Basically, we can assume objective function based only on the area of prestressing reinforcement in one section (see formula 6.1). A major disadvantage is in not covering length of the tendon in objective function. Hereby, the optimization is partly distorted. A typical case is explained in Chapter 7.2 in details.

$$f(x) = \min\left(A_{\mathrm{p}}\right) = \sum_{\mathrm{i}=1}^{\mathrm{n}} A_{\mathrm{p,i}} \cdot n_{\mathrm{t,i}} \cdot n_{\mathrm{g,i}}. \tag{7.1}$$

2) Mass of prestressing

The extended solution covers also length of the tendons. The cross-sectional area and length of each tendon define the total mass of the post-tensioned tendons $\left(M_{\mathrm{p}}\right)$. An illustrative example is mentioned in 7.4. The objective function can be written as follows

$$f(x) = \min\left(M_{\mathrm{p}}\right) = \sum_{\mathrm{i}=1}^{\mathrm{n}} A_{\mathrm{p,i}} \cdot n_{\mathrm{t,i}} \cdot n_{\mathrm{g,i}} \cdot L_{\mathrm{p,i}} \cdot \gamma_{\mathrm{p,i}}. \tag{7.2}$$

3) Total cost of bridge

The last and the most relevant case is the optimization based on the total cost of the structure. Here, cost for concrete and post-tensioning give the total material cost of the structure. A formula representing this case can be expressed in the following formula

$$f(\mathrm{x}) = \min(C_{\mathrm{m}}) = C_{\mathrm{p}} \cdot \left(\sum_{\mathrm{i}=1}^{\mathrm{n}} A_{\mathrm{p,i}} \cdot n_{\mathrm{t,i}} \cdot n_{\mathrm{g,i}} \cdot L_{\mathrm{p,i}}\right) + C_{\mathrm{c}} \cdot A_{\mathrm{c}} \cdot L_{\mathrm{tot}}. \tag{7.3}$$

The items in formulas above mean: $A_{\mathrm{p,i}}$ is the area of one strands Y1770S7-15.7; values $n_{\mathrm{t,i}}$ and $n_{\mathrm{g,i}}$ are a numbers of strands in tendon *i* and a number of the same tendons, respectively; $L_{\mathrm{p,i}}$ is a length of the particular tendon; $\gamma_{\mathrm{p,i}}$ is an unity mass of the prestressing tendons; $A_{\mathrm{c}}$ is a area of concrete cross-section; $L_{\mathrm{tot}}$ is a total length of the bridge; $C_{\mathrm{p}}$ and $C_{\mathrm{c}}$are the cost for concrete and prestressing steel unit defined in 6.5.1, respectively; $i = 1 \dots n$ where $n$ is a number of particular tendon geometry.

### 7.1.4 Constraints

Usually each optimization task requires at least one constraint in practice (constrained optimization problem). This can be on side of limit range of the variables too (see Chapter 7.1.2.4). The concrete bridges have to satisfy many kinds of design checks (ULS, SLS, detailing provision, etc.). But not all of them must be necessarily included in the optimization in order to prevent too time-consuming calculation. The particular design codes require slightly different approaches of them. We focus on the design of structure based on the Eurocodes. Thus, all checks are presented based on Eurocodes assumption. The most efficient constraints are presented in the next text.

- Allowable concrete stresses

All codes verify minimal compressive and maximal tensile concrete stress as follows

$$g_1(x) = \frac{\sigma_{cc}}{\sigma_{cc,lim}} - 1 \leq 0, \quad (7.4)$$

$$g_2(x) = \frac{\sigma_{ct}}{\sigma_{ct,lim}} - 1 \leq 0. \quad (7.5)$$

where the concrete stresses in the cross-section are based on the elastic theory calculated according to following formula

$$\sigma_{cc(ct)} = \frac{N}{A} \pm \frac{M_y}{W_z} \quad (7.6)$$

The limits for tensile and compressive strength depend on the combination and construction process. During the optimization, the following list of strength can be verified.

  - Strength before and after anchoring – defined in formula 5.42 from [133]_as follows

$$\sigma_{cc,max} = k_6 \cdot f_{ck}(t). \quad (7.7)$$

  - Strength resisting to longitudinal cracks – defined for characteristic combination_in Chapter 7.2(2) from [133] as follows

$$\sigma_{cc,ch} = k_2 \cdot f_{ck}(t). \quad (7.8)$$

  - Strength assuming liner creep behaviour – defined for quasi-permanent combination_in Chapter 7.2(3) from [133] as follows

$$\sigma_{cc,ch} = k_2 \cdot f_{ck}(t). \quad (7.9)$$

Note: Values $k_1$, $k_2$ and $k_6$ defined in the particular chapters of [133] have defaults 0.6, 0.45 and 0.6, respectively.

- Cross-section capacity

Additionally, the checks of cross-section in ultimate limit state (ULS) also significantly affect the optimization problem. The normal force and bending moment capacity in ULS can be verified by the interaction diagram method,

$$g_3(x) = \frac{N_{Ed}}{N_u} - 1 \leq 0, \quad (7.10)$$

$$g_4(x) = \frac{M_{Ed}}{M_u} - 1 \leq 0, \quad (7.11)$$

Typical interaction diagram is presented in Fig. 7.4. An iterative process finds the interaction surface representing maximal allowed values of normal force and bending moments which the cross-section can carry.

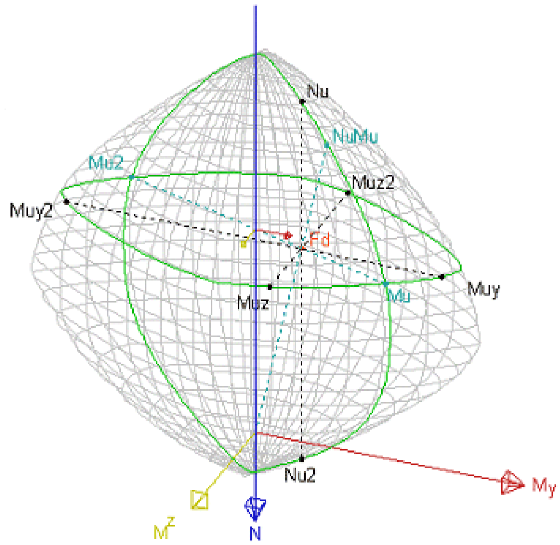



Fig. 7.4 Interaction diagram

The second possibility represents the calculation of strain in concrete ($\varepsilon_{cc}$) and prestressing tendon ($\varepsilon_{tt}$) and comparison with limits strains of particular cross-section components as follows

$$g_5(x) = \frac{\varepsilon_{cc}}{\varepsilon_{cu}} - 1 \leq 0, \tag{7.12}$$

$$g_6(x) = \frac{\varepsilon_{tt}}{\varepsilon_{ud}} - 1 \leq 0. \tag{7.13}$$

The basic assumptions of this limit strain method shows Fig. 7.5. For interested readers we recommend the publication [92] with more details. Generally, four limit strain states can occur. The numbering (1-4) in Fig. 7.5 represents particular state types of the cross-section. The state (1) corresponds to the optimal failure when ultimate compressive strain in concrete ($\varepsilon_{cu}$) and ultimate tensile strain in prestressing ($\varepsilon_{ud}$) are reached. In case of state (2), the ultimate limit strain in concrete is assumed within considering the strain in prestressing at the beginning of plastic branch ($\varepsilon_{pe}$). The state (3) expresses the starting of the concrete crushing. Finally, the state (4) represents the reaching of ultimate compressive strain for axially loaded member decreased due to brittle failure effect.

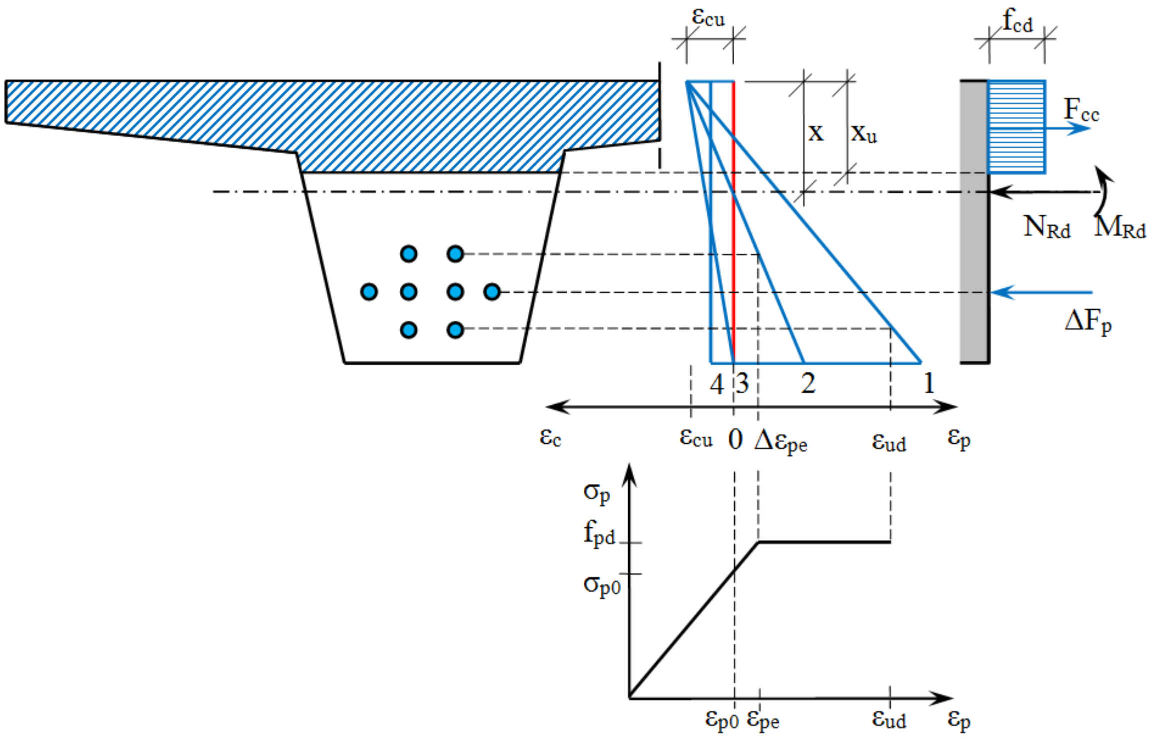



Fig. 7.5 Limit strain method

- <u>Allowable stresses of prestressing</u>

Due to tendon losses, the prestressing force is not constant in time. Eurocode requires a verification of calculated stress ($\sigma_p$) in prestressing tendon with its limits ($\sigma_{p,lim}$).

$$g_7(x) = \frac{\sigma_p}{\sigma_{p,lim}} - 1 \leq 0, \tag{7.14}$$

The constraint includes a check of prestressing prior ($\sigma_{pa}$), after ($\sigma_{aa}$) anchoring and strength resisting to undesirable cracks and deformations ($\sigma_{pm}$). The limits are described in details in the following formulas

- <u>Strength prior anchoring</u> – defined in formula 5.41 from [133] as follows

$$\sigma_{pa} = \min(k_1 \cdot f_{pk}; k_2 \cdot f_{p0,1k}). \tag{7.15}$$

- <u>Strength after anchoring</u> – defined in formula 5.43 from [133] as follows

$$\sigma_{aa} = \min(k_7 \cdot f_{pk}; k_8 \cdot f_{p0,1k}). \tag{7.16}$$

- <u>Strength resisting to undesirable cracks and deformations</u> - defined in Chapter 7.2(5) from [133] as follows

$$\sigma_{pm} = \min(k_5 \cdot f_{pk}). \tag{7.17}$$

Note: Values $k_1$, $k_2$, $k_5$, $k_7$ and $k_8$ defined in the particular chapters of [133] have defaults 0.8, 0.9, 0.75, 0.75 and 0.85, respectively.

- Shear capacity

There exists general concept of “strut-and-tie” model [133]for the prediction of shear effects in concrete, see Fig. 7.10. In this model, the top compression and bottom tensile members represent the compressive concrete and tensile reinforcement, respectively. The horizontal members are connected by the compressive virtual struts and reinforcement tensile ties. The axial forces in tensile ties should be transmitted by the shear reinforcement. Consequently, the maximal force in concrete struts ($V_{\mathrm{Rd,max}}$) and shear force retained by the shear resistance ($V_{\mathrm{Rd,s}}$) have to be compared with acting shear force ($V_{\mathrm{Ed}}$), see following formulas

$$g_8(x) = \frac{V_{\mathrm{Ed}}}{V_{\mathrm{Rd,max}}} - 1 \leq 0, \tag{7.18}$$

$$g_9(x) = \frac{V_{\mathrm{Ed}}}{V_{\mathrm{Rd,s}}} - 1 \leq 0. \tag{7.19}$$

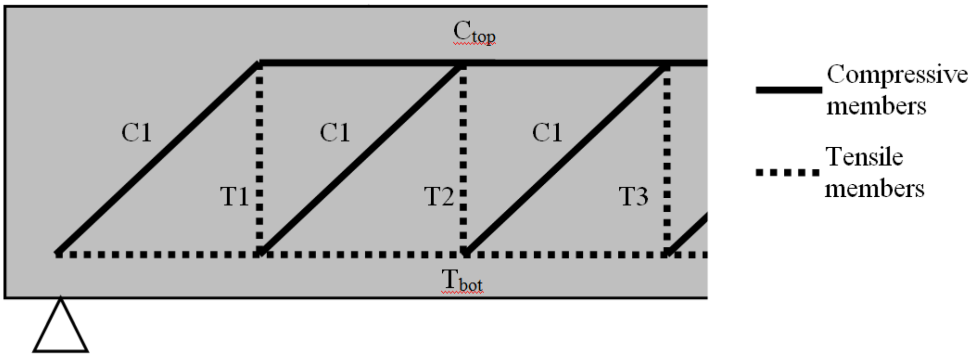



Fig. 7.6 Strut-and-tie model

However, the strut and tie model is required, the post-tensioned concrete bridges have many times a massive cross-section. A non-prestressed reinforcement is often considered as a minimal required only. Therefore, the shear capacity of concrete $(V_{\mathrm{Rd,c}})$ is usually crucial in ULS and can be expressed as formula 6.2.2 from [133]

$$g_{10}(x) = \frac{V_{\mathrm{Ed}}}{V_{\mathrm{Rd,c}}} - 1 \leq 0. \tag{7.20}$$

- Fatigue check

A fatigue action on bridges is very dangerous due to variation of the variable load. The changes of the load levels repeat in many loading cycles. The verification is important for the relatively subtle structure with long spans. In this case, the stress range of the concrete ($\Delta\sigma_{\mathrm{c,fat}}$) and prestressing steel ($\Delta\sigma_{\mathrm{p,fat}}$) is compared with allowable stress limits ($\Delta\sigma_{\mathrm{c,fat,lim}}; \Delta\sigma_{\mathrm{p,fat,lim}}$). Usually, the concrete represents a more exposed component of the fatigue actions. The different behaviour can appear in cable-stayed or suspension structures where the cables are subjected to extremely different stresses from the variable load,

$$g_{11}(x) = \frac{\Delta\sigma_{\mathrm{c,fat}}}{\Delta\sigma_{\mathrm{c,fat,lim}}} - 1 \leq 0, \tag{7.21}$$

$$g_{12}(x) = \frac{\Delta\sigma_{\mathrm{p,fat}}}{\Delta\sigma_{\mathrm{p,fat,lim}}} - 1 \leq 0. \tag{7.22}$$

- Detailing provisions

All constraints mentioned above relate to verification of the “stress state”. The internal forces, stresses are compared with limit values. The second group focuses on detailing provisions of the prestressing. Therefore, the minimal distances between the tendon ducts according to EN1992-1-1 [133] (see Fig. 7.7) and minimal concrete cover for post-tensioned reinforcement have to be satisfied. Practically, the minimal concrete cover is ensured by the minimal or maximal range of the variable.

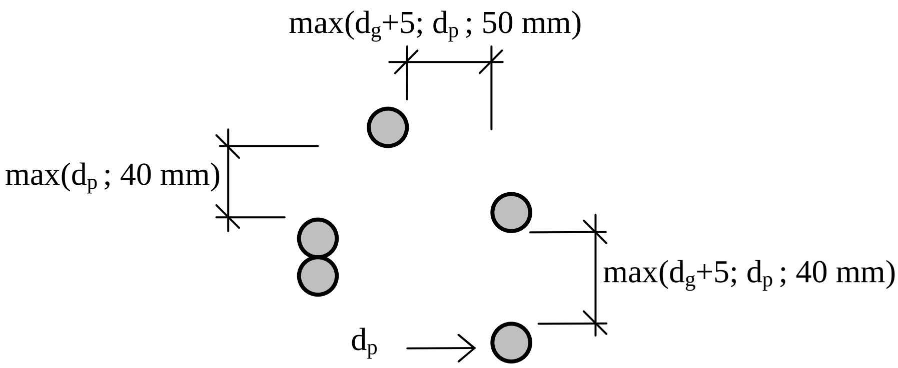



Fig. 7.7 Detailing provisions of post-tensioned reinforcement according to EN1992-1-1

As has been summarized, there exist many constraints for optimal design of post-tensioned concrete bridges. It is suitable to select the most critical constraints and use them during the optimization. Obviously, the resulting optimal structure can be run manually for all additional checks.

### 7.1.5 Selection of the mathematical algorithm for optimization

There are many mathematical methods for optimization of structures (see Chapter 4). It is almost impossible to declare one algorithm as the general one for optimization of any kind of the structure. As was mentioned before, there are many reasons influencing the selection of the most suitable method.

Generally, the number of tendons is an integer independent variable. Therefore, a method enabling calculation within integer parameters can be selected only. Two implement methods fulfil this criterion. The first method of differential evolution is usable.

Nevertheless, several instances of testing confirm not effective convergence. The method of modified simulated annealing, as the second possibility, proves all expected requirements. The method gets relatively fast convergence with good enough solution. In some cases, it is recommended to improve an optimum found using MSA method by the more accurate method of sequential quadratic programming (see Fig. 7.8).

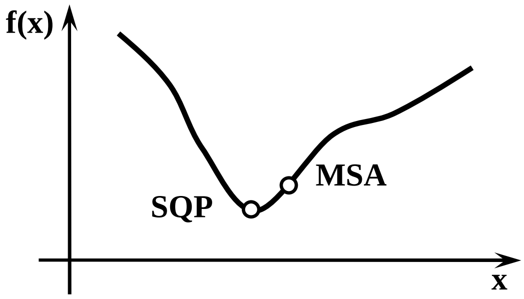



Fig. 7.8 Quality of the optimum based on the used method

### 7.1.6 Post-processing analysis and verification of the optimized results

Let us assume a finalization of optimization process. It can be typically reached by convergence criterion or maximal number of iterations. In case of MSA method, the optimization process finds group of optimal solution. Afterwards, a post-processing analysis should be done by designer. Just now, it is his responsibility to select correct one from provided solutions by optimization process. In some cases, the constraints do not include all limitations and it is recommended to verify optimal solutions manually. Typically, some optimum based on allowable concrete stresses are refused due to not satisfying capacity of cross-section in ultimate limit state. Additionally, there can be requirements based on the technology or aesthetic, which should be applied too.

### 7.1.7 Optimized post-tensioned concrete bridges

The previous chapter dealt with the procedure of optimization of post-tensioned concrete bridges. The basic requirements and limitations were discussed. Now, let us consider three different post-tensioned concrete bridges where slightly different objective functions are used. The bridges with particular different cross-sections, number of spans and theirs lengths have the different process of construction. The following three examples will be explained in the next text.

- <u>Three girders three spans bridge</u> – bridge cast-in-place in fixed formwork, the objective function is based on the area of prestressing reinforcement.

- Double T-section four span post-tensioned bridge – bridge built using span-by-span technique with cantilever overhang focuses on optimization of total material bridge cost.
- Nine span deck bridge – the most complex example including several construction stages requiring deeper analysis, a total mass of post-tensioned tendons is optimized in this case

## 7.2 Three girders three spans post-tensioned bridge

This chapter presents example of the optimum design of a tendon geometry of three spans post-tensioned concrete structure investigated up to necessary details with occurrence of discrete variables. A post-tensioned concrete three spans (28+36+28 m) bridge is optimized from the prestressing level as well as a geometry point of view. The cross-sectional shape is a three-beam with fixed dimensions of a depth (1.865 m) and bottom width of beams equal to 1.2 m. The upper part of deck is 16.55 m wide. The shape of cross-section is shown in Fig. 7.9. A structure is built from concrete C35/45. The design and check of the structure is performed according to Eurocodes with respect to Czech national annex.

The load combinations are considered according to ČSN EN 1990 with respect of A2 attachment. The structure is loaded by variable traffic load according to ČSN EN1991-2 where load group gr1a (a tandem system, uniform dead load and pedestrians) seems to be dominant. The envelopes of bending moments and shear forces from gr1a were evaluated during analysis. The temperature changes of the structure (linear heating and cooling) together with initial support displacements were also considered in the calculation. The structure is analyzed using construction stages with time dependent analysis of creep and shrinkage behaviour again according to annex B of ČSN EN 1992-1-1. As usual, the structure is designed for 100 years.

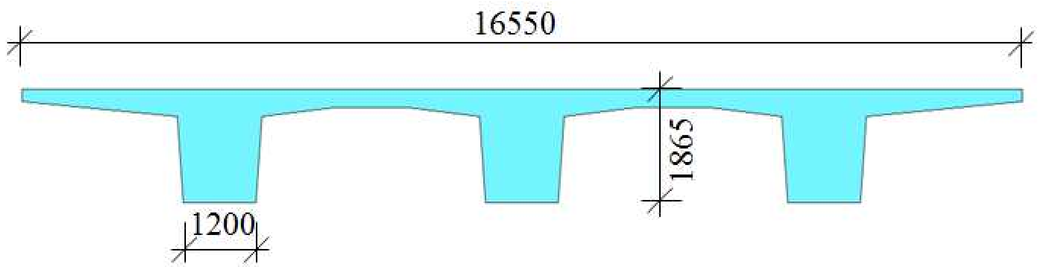



Fig. 7.9 A bridge cross-section (in mm)

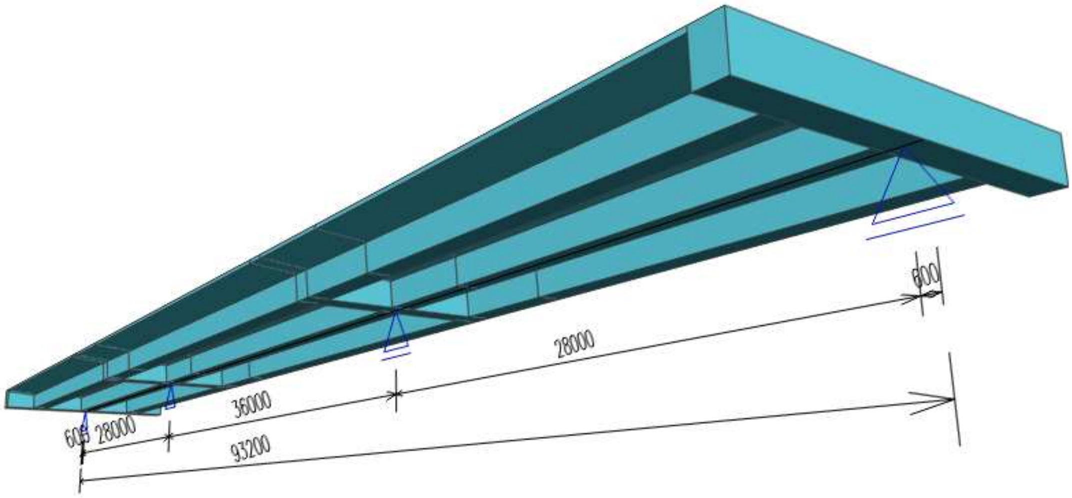



Fig. 7.10 A structural model of bridge (in mm)

### 7.2.1 Transversal spreading of the load

Because of the three beams cross-section the transversal spreading load was performed at first. The model for transversal spreading was 2D model, where the concrete 2D slab was modelled as deck. Each beam was connected to the slab like a rib. Particularly, the internal beam takes 45.5 % and the edge beam 53.3 % of the acting variable load. Only one cross-section was studied for the next analysis. A typical studied cross-section is displayed in Fig. 7.11. This is the edge beam which takes 53.3 % from the load system of a variable mobile load.

Tab. 7.1 Transversal spreading of the load

| Beam | 1 | 2 | 3 | Sum |
|---|---|---|---|---|
| Edge | 3624 kNm | 2290 kNm | 831 kNm | 6745 kNm |
| | 53.3 % | 34.0 % | 12.7 % | 100 % |
| Middle | 1823.5 kNm | 3098 kNm | 1823.5 kNm | 6745 kNm |
| | 27.25 % | 45.5 % | 27.25 % | 100 % |

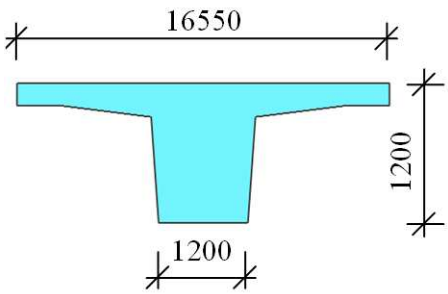



Fig. 7.11 A typical cross-section of one beam (in mm)

### 7.2.2 Objective function

The prestressing of one beam consists of different tendon geometries A-E as shown in Fig. 7.12. The aim of this study was to minimize a necessary area of prestressing reinforcement together with more effective tendon geometry. The same number of strands was assumed in tendon geometries A, B, C and another in the groups of tendons D and E. Objective function can be expressed as follows

$$f(x) = \min\left(A_{\mathrm{p}}\right) = \sum_{\mathrm{i}=1}^{\mathrm{n}} A_{\mathrm{p,i}} \cdot n_{\mathrm{t,i}} \cdot n_{\mathrm{g,i}}. \tag{7.23}$$

where $A_{\mathrm{p,i}}$ is the area of one strands Y1770S7-16.0; values $n_{\mathrm{t,i}}$ and $n_{\mathrm{g,i}}$ are a numbers of strands in tendon $i$ number and a number of tendons in a group for one beam, respectively; $i = 1 \dots n$ where $n$ is a number of particular tendon geometry.

The length of the tendon varies according to optimized geometry. Authors also performed a study where these lengths were included in the objective function. Nevertheless, the results gave almost negligible decreasing of the total length of the tendon. There are more than 10 km of strands and a difference just only about 5 meters among optimized solution was obtained. The area of prestressing reinforcement in case of a continuous tendon geometry on the whole bridge seems to be the most important factor. Therefore, the effects of the length of the tendon geometry can be neglected.

### 7.2.3 Design variables

The numbers of strands in one tendon and the number of tendons in group were selected as design variables. There are five different geometries in the one beam (A-E). The number of the same tendon in group was constant for geometry A-C. Particularly six tendons in a cross-section were selected, i.e. two tendons in one beam.

Tab. 7.2 Initial and limit values for design variables

| Parametr | $n_{\mathrm{tABC}}$ [-] | $n_{\mathrm{tDE}}$ [-] | $n_{\mathrm{gDE}}$ [-] | $x_{\mathrm{A1}}$ [m] | $z_{\mathrm{A1}}$ [m] | $z_{\mathrm{B1}}$ [m] | $z_{\mathrm{A2}}$ [m] | $x_{\mathrm{A4}}$ [m] |
|---|---|---|---|---|---|---|---|---|
| Initial | 15 | 15 | 2 | 19.1 | 0.15 | 1.35 | 1.4 | 38.1 |
| Minimum | 15 | 15 | 0 | 17.1 | 0.15 | 0.45 | 1.25 | 36.6 |
| Maximum | 19 | 15 | 2 | 21.1 | 0.45 | 1.35 | 1.55 | 39.6 |
| Step | 2 | - | 1 | 0.5 | 0.05 | 0.1 | 0.05 | 0.5 |

Next, the number of strands in tendons for the D-E geometries was fixed to 15. Therefore, we have two independent design variables connected to the prestressing area, namely the number of strands for the A-C geometries $n_{t,ABC}$ and the number of groups for the D-E geometries $n_{g,DE}$, see Tab. 7.2 for initial values and predefined limits.

Other design variables are geometry coordinates of tendon geometry. Each geometry of tendon has 4 points which positions are optimized (marked red in Fig. 7.12) and are again listed in Tab. 7.2. The remaining points of geometry were calculated based on these design variables. Dependency between all points of geometry decreases the number of geometry design variables to 7 only. The dependent variables are summarized in the following table (see Tab. 7.3). Values $x_{step}$=2.0m and $x_{step1}$=1.5m and $z_{step}$=0.15m are kept constant.

Tab. 7.3 Dependent parameters

| Variable | Formula | Variable | Formula |
|---|---|---|---|
| $z_{A3}$ | $z_{A2}$ | $x_{B5}$ | $L$ - $x_{B4}$ |
| $x_{B4}$ | $x_{A4} + x_{step}$ | $x_{C5}$ | $L$ - $x_{C4}$ |
| $z_{B2}$ | $z_{A2} + z_{step}$ | $z_{B1pom}$ | $(z_{B2} - 1.1) \cdot 9.6/(L_1 - 1) + 1.1$ |
| $z_{B3}$ | $z_{B2}$ | $z_{C1pom}$ | $(z_{C2} - 1.5) \cdot 11.1/(L_1 - 1) + 1.5$ |
| $z_{B4}$ | $z_{A4} + z_{step}$ | $x_{B1}$ | $x_{A1}$ - $x_{step1}$ |
| $x_{C4}$ | $x_{B4} + x_{step}$ | $x_{C1}$ | $x_{B1}$ - $x_{step1}$ |
| $z_{C2}$ | $z_{B2} + z_{step}$ | $z_{C1}$ | $z_{B1} + z_{step}$ |
| $z_{C3}$ | $z_{C2}$ | $x_{B8}$ | $L$ - $x_{B1}$ |
| $z_{C4}$ | $z_{B4} + z_{step}$ | $x_{C8}$ | $L$ - $x_{C1}$ |
| $L$ | $2 \cdot L_1 + L_2$ | $z_{B0}$ | $(z_{B1} < 1.1) \cdot z_{B1} + (1.1 \leq z_{B1}) \cdot z_{B1pom}$ |
| $x_{A5}$ | $L$ - $x_{A4}$ | $z_{C0}$ | $(z_{C1} < 1.5) \cdot z_{C1} + (1.5 \leq z_{C1}) \cdot z_{C1pom}$ |
| $x_{A8}$ | $L$ - $x_{A1}$ | | |

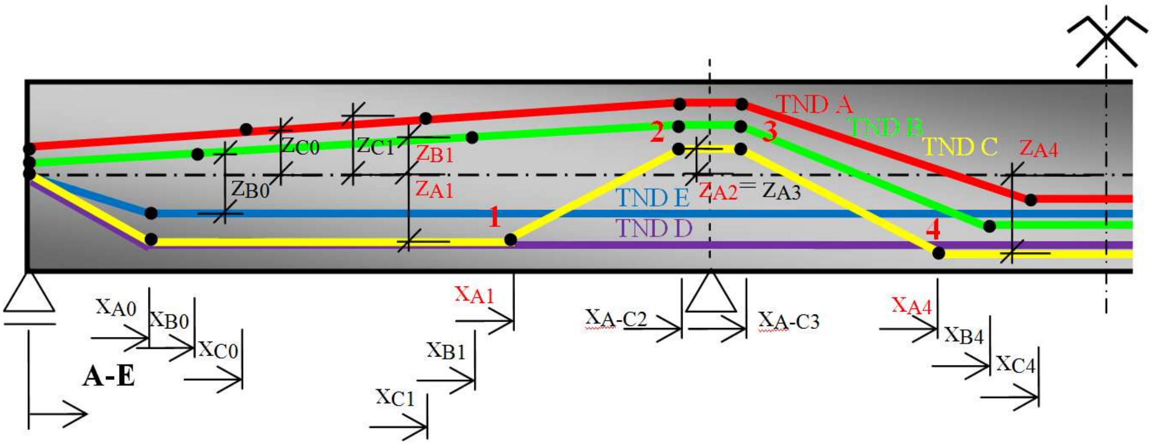



Fig. 7.12 Tendon geometry of post-tensioning. Design variables are marked red.

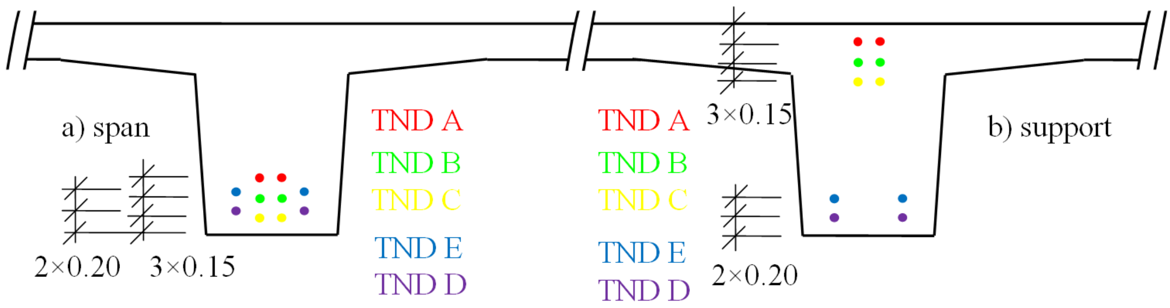



Fig. 7.13 Distribution of the tendons in the cross-section –a) span, b) support

### 7.2.4 Constraints

The optimization of prestressing reinforcement was performed based on the serviceability limit state (crack appearance using check of allowable concrete stresses from the characteristic combination) and on the ultimate limit state (a check of capacity calculated using an interaction diagram for acting combination of a normal force and a bending moment). The combinations were evaluated for 100 years of bridge service. Limit values of checks (a ratio of calculated and limit values) was selected in a standard way 1.0. The used constraints can be divided into three groups.

- Geometrical – these constraints are coming from a geometry of cross-section and distribution of the individual tendons in the cross-section with respect to a minimal concrete cover and clear distances between tendons.
- Serviceability limit state (Check A) – normal stresses for the characteristic combination during service in 100 years are evaluated as the indication of the longitudinal cracks existence. The maximal and minimal stress in concrete is compared with allowable concrete stress in tension $\left(f_{\mathrm{ct,eff}} = 3.76\ \mathrm{MPa}\right)$ and in compression$\left(\sigma_{\mathrm{cc,ch}} = 0.6\ f_{\mathrm{ck}} = 21\ \mathrm{MPa}\right)$,

$$g_1(x) = \frac{\sigma_{\mathrm{cc}}}{\sigma_{\mathrm{cc,ch}}} - 1 \leq 0, \tag{7.24}$$

$$g_2(x) = \frac{\sigma_{\mathrm{ct}}}{\mathrm{f_{ct,eff}}} - 1 \leq 0. \tag{7.25}$$

- Ultimate limit state (Check B) – a verification of the beam loaded by the combination of a normal force and a bending moment is performed by a method of the interaction diagram. Fundamental STR/GEO Set B combination was used for verification in this check,

$$g_3(x) = \frac{N_{\mathrm{Ed}}}{N_{\mathrm{u}}} - 1 \leq 0, \tag{7.26}$$

$$g_4(x) = \frac{M_{\mathrm{Ed}}}{M_{\mathrm{u}}} - 1 \leq 0. \tag{7.27}$$

### 7.2.5 Optimization algorithm and results

There exist many optimization algorithms which can be used for optimization (see Chapter 4). For our case where number of strands and tendons are discrete design variables it is necessary to use method which is capable to handle discrete variables. Evolutionary algorithms are a group of optimization methods that mimic the evolution of nature in aim to search optima. Modified simulated annealing (MSA) or differential evolution (DE) are well-known examples of these methods. We tested both method and MSA has been finally selected due to faster convergence.

The MSA is characterized by the population of candidate solutions, where the population consisted of 15 members. This method allows to set changing step of design variables. The simulated annealing algorithm is presented in the selection phase, where the acceptance of new solutions is governed by the cooling algorithm. Here, the initial annealing temperature was calculated based on an acceptance of 50% of members from the first population leading to $T_{max}$ = 6686.24. Then the cooling coefficient was set to $T_{mult}$ = 0.736. Note that all solutions are presented in Fig. 7.14, i.e. including solutions that do not fulfil given constraints. The total optimization time was 20hrs and 50mins which is still relatively acceptable.

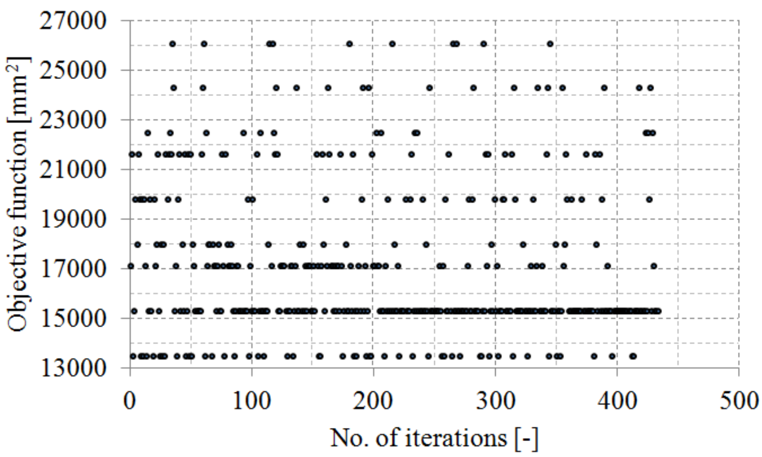



Fig. 7.14 Value of objective History of the objective function development.

As the results, 9 possible solutions were offered by the program after running 484 iterations. Based on the detailed upcoming analysis we can accept solution 7, 8 and 9 characterized by the 19 strands in a tendon. Tendons with geometry D and E are not necessary at all. When we use tendons with 17 strands we received ratio of acting design bending moment (calculated value) to cross-section bending resistance (limit value) in range 1.003–1.007, which are slightly out of the limit range. The ratios of calculated stresses to allowable concrete stresses and design acting forces to cross-section resistance calculated by interaction diagram tend to limit value 1.0. Maximal calculated ratios are presented in table Tab. 7.5. All obtained results were also verified on other checks for prestressed concrete according to ČSN EN 1992-2. Allowable concrete stresses for characteristic and frequent combinations, allowable stresses in prestressing reinforcement prior and after anchoring and shear verification were performed as well. Origin tendon geometry and a new optimized geometry are compared on the half of the secure in Fig. 7.15. Vertical geometries of the tendons together with cross-section are five times scaled to highlight the differences. The saving of material is compared for particular solution together with their ratios of check in Tab. 7.5.

Tab. 7.4 Optimized solution found by MSA method

| Parameter | $n_{t,ABC}$ [-] | $n_{t,DE}$ [-] | $n_{g,DE}$ [-] | $x_{A1}$ [m] | $z_{A1}$ [m] | $z_{B1}$ [m] | $z_{A2}$ [m] | $x_{A4}$ [m] | $z_{A4}$ [m] | $A_{p,req}$ [mm$^2$] |
|---|---|---|---|---|---|---|---|---|---|---|
| Initial | 15 | 15 | 2 | 9.1 | 0.15 | 1.35 | 1.4 | 38.1 | 0.15 | 22500 |
| Sol. 1 | 17 | 15 | 0 | 17.1 | 0.4 | 1.25 | 1.4 | 39.1 | 0.45 | 15300 |
| Sol.2 | 17 | 15 | 0 | 17.1 | 0.4 | 1.35 | 1.4 | 38.6 | 0.40 | 15300 |
| Sol.3 | 17 | 15 | 0 | 17.1 | 0.25 | 1.25 | 1.4 | 38.1 | 0.45 | 15300 |
| Sol.4 | 17 | 15 | 0 | 18.1 | 0.4 | 1.25 | 1.4 | 39.6 | 0.45 | 15300 |
| Sol.5 | 17 | 15 | 0 | 17.6 | 0.25 | 1.35 | 1.4 | 39.1 | 0.45 | 15300 |
| Sol.6 | 17 | 15 | 0 | 17.6 | 0.35 | 1.25 | 1.4 | 39.1 | 0.40 | 15300 |
| Sol.7 | 19 | 15 | 0 | 18.6 | 0.35 | 1.15 | 1.4 | 38.1 | 0.35 | 17100 |
| Sol.8 | 19 | 15 | 0 | 17.6 | 0.45 | 1.1 | 1.4 | 38.6 | 0.40 | 17100 |
| Sol.9 | 19 | 15 | 0 | 17.6 | 0.2 | 0.55 | 1.4 | 37.6 | 0.35 | 17100 |

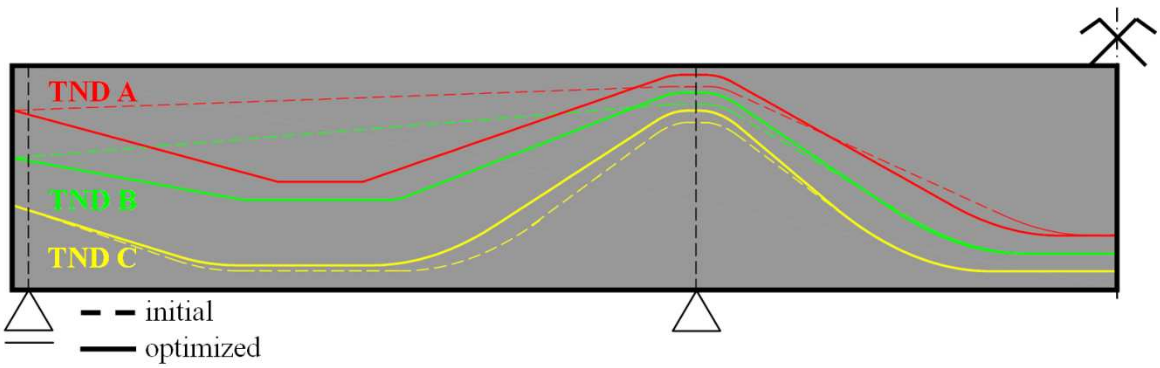



Fig. 7.15 Comparison of initial and optimized geometry

Tab. 7.5 Results for offered solutions.

| Parameter | Check A [-] | Check B [-] | $A_{p,req}$ [mm$^2$] | Save [%] |
|---|---|---|---|---|
| Initial | 0.696 | 0.856 | 22500 | - |
| Sol. 1 | 0.710 | 1.004 | 15300 | 32 |
| Sol. 2 | 0.701 | 1.005 | 15300 | 32 |
| Sol. 3 | 0.685 | 1.007 | 15300 | 32 |
| Sol. 4 | 0.711 | 1.003 | 15300 | 32 |
| Sol. 5 | 0.709 | 1.003 | 15300 | 32 |
| Sol. 6 | 0.687 | 1.007 | 15300 | 32 |
| Sol. 7 | 0.680 | 0.950 | 17100 | 24 |
| Sol. 8 | 0.664 | 0.945 | 17100 | 24 |
| Sol. 9 | 0.778 | 0.970 | 17100 | 24 |

The MSA method decreased amount of prestressing reinforcement by 24 % by modifying the tendon geometry. The ratio of allowable concrete stresses check for characteristic combination and check of capacity using interaction diagram are very close to limit ratio 1.0, see Fig. 7.16.

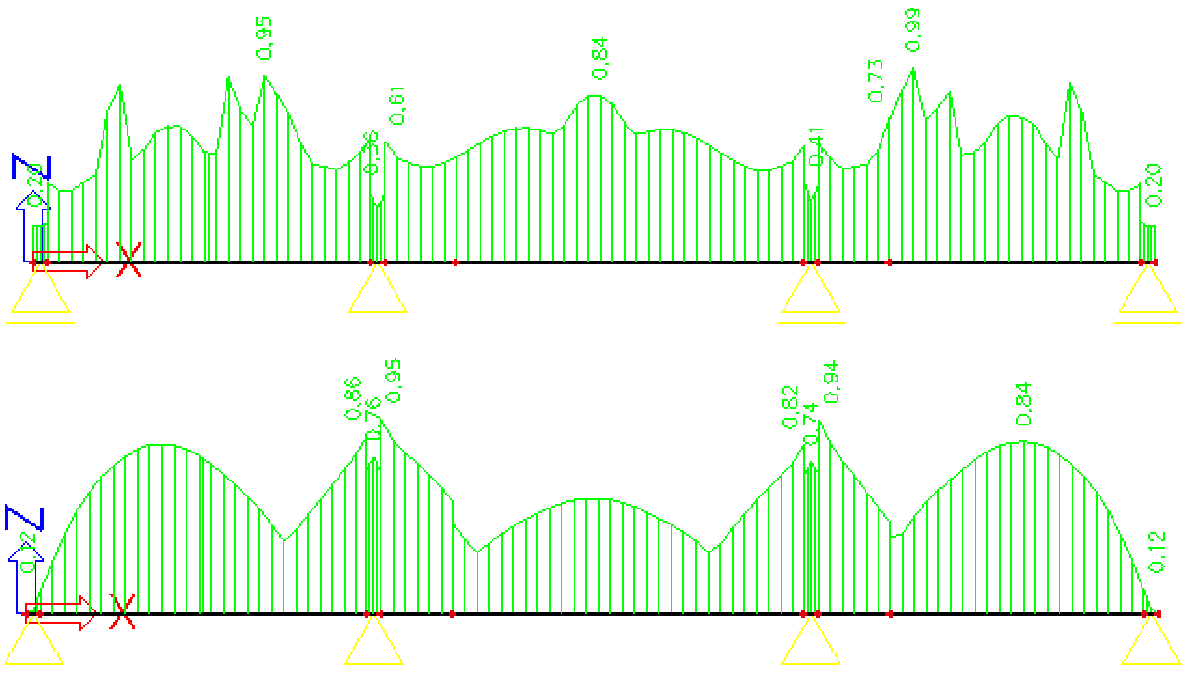



Fig. 7.16 Ratio for allowable concrete stresses check (top) and capacity check using interaction diagram (bottom) in 100 years for solution no.7

The results for solution 4 are mentioned in the next figure (see Fig. 7.17) for illustration purposes. Here, the capacity check using interaction diagram is not satisfied only with 0.3 %.

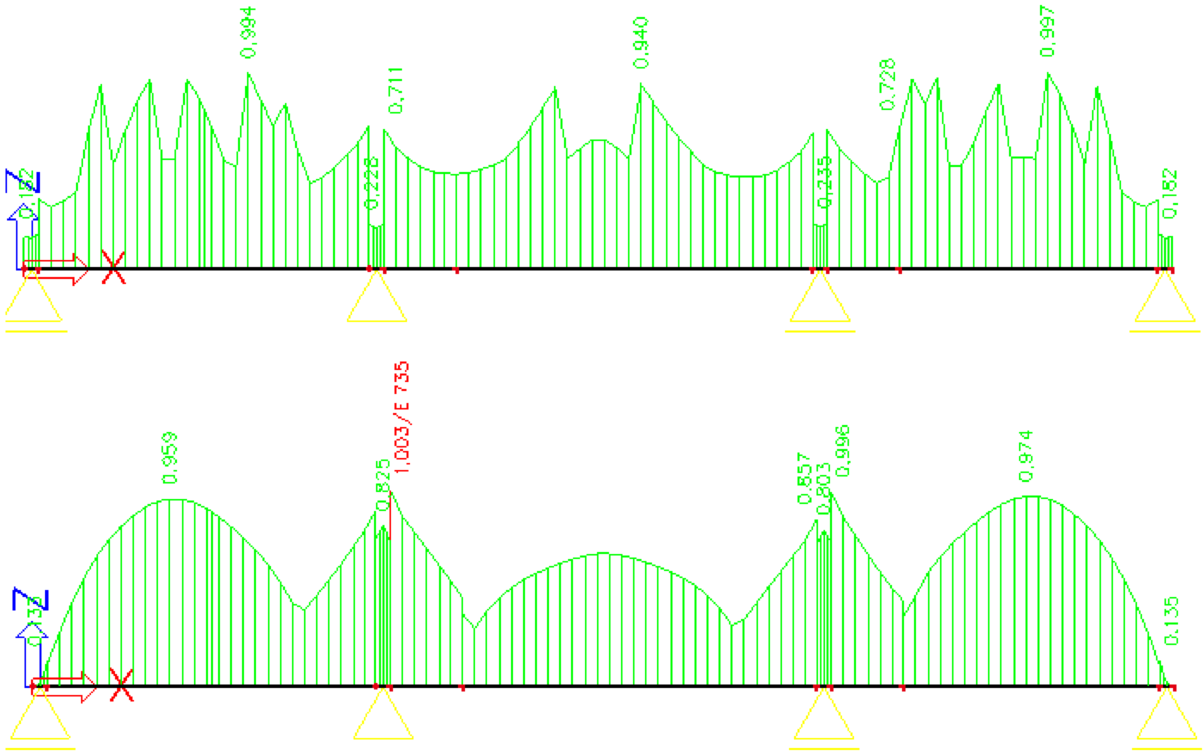



Fig. 7.17 Ratio for allowable concrete stresses check (top) and capacity check using interaction diagram (bottom) in 100 years for solution no. 4

From the obtained results we can conclude it is possible to used 6 pieces of 19 strands of geometries A-C and that tendons with geometries D and E are not necessary at all. Since the used code includes many safety factors, it is also possible to use the solution 4 violating constraints only by 0.3%. Because the numbers of strands are discrete design variables, it was necessary to apply an optimization method which is able to handle the discrete type of parameters. Here, Modified Simulated Annealing was successfully used. The obtained amount of prestressing reinforcement was decreased by 24% in comparison to the original design.

## 7.3 Double T-section four span post-tensioned bridge

The typical example of bridge used very often in practice is post-tensioned double T-section bridge. Therefore, the application of the optimization methods has been demonstrated on this kind of structure. A bridge cross-section from C35/45 is displayed in Fig. 7.18.

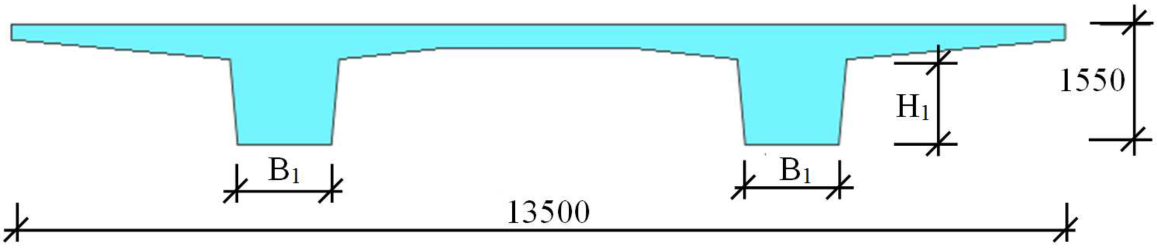
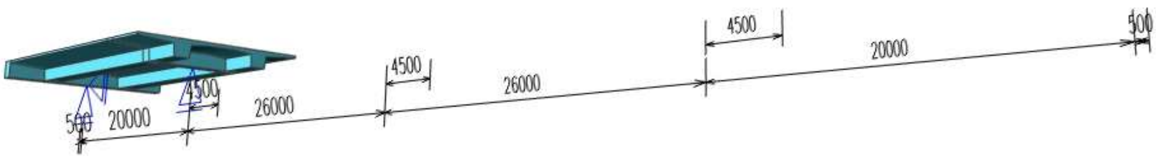


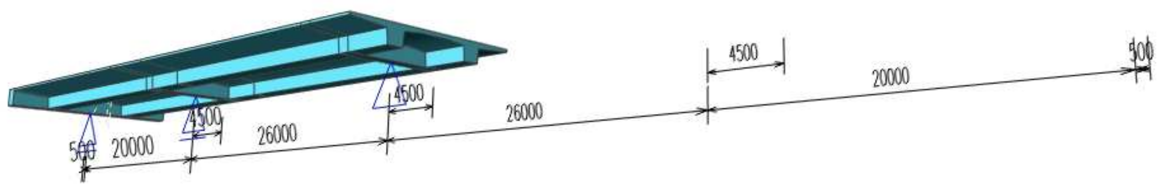

Fig. 7.18 Double T-section

The bridge consisting of four spans (20+26+26+20 m) is built using technique span-by-span with overhang cantilever. A total length of the bridge including edge beams above the supports is $L_{tot} = 93.0$m. The construction of each span includes two phases. At first, the concrete is casted-in-place to fixed formwork and next prestressing of the span is applied. This procedure repeats for each span and completely 4 groups of the construction stages are modelled. The prestressing is applied incrementally across two spans. The couplers connect the half of the tendons in vertical construction joint of each cantilever overhang. A group of four figures Fig. 7.19 shows the construction procedure of the bridge.

**Stage 1**




**Stage 2**



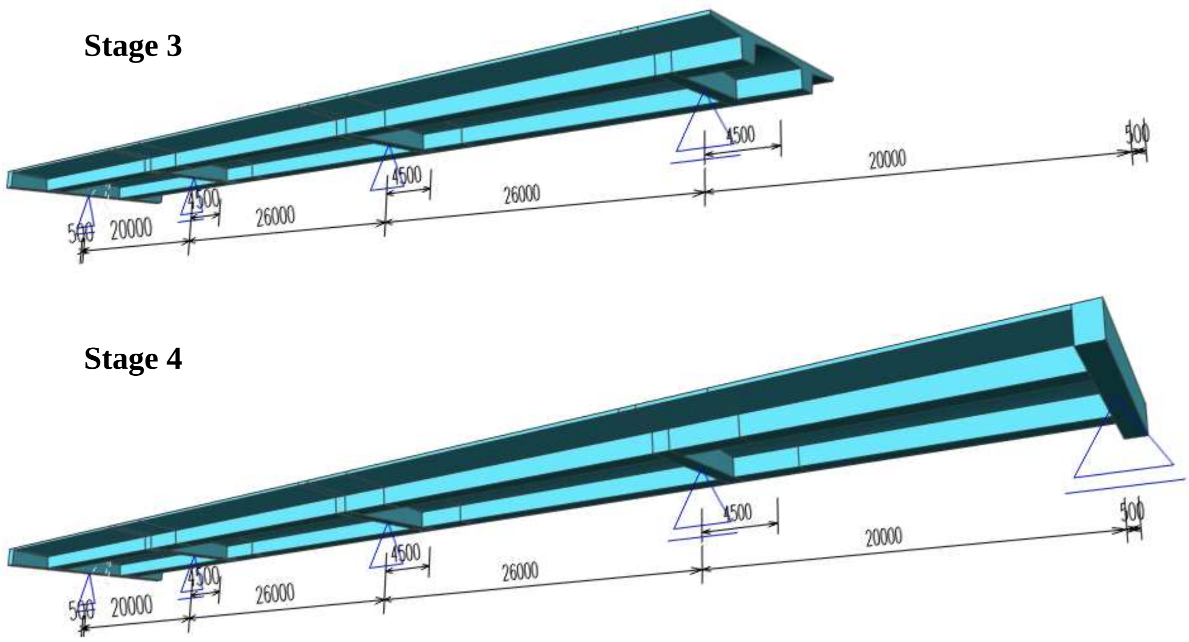



Fig. 7.19 Construction process of the bridge

The Eurocodes are valid in Czech Republic from 03/2010. Therefore, the bridge has to satisfy all requirements coming from particular Eurocodes. The structure analysis takes into account also an effect of span-by-span construction. Simultaneously, the rheological behaviour concrete creep and shrinkage is covered in the model of CEB-FIP 1990, which is included in EN1992-1-1. It belongs to the most complex models for time analysis classified to "product models". This group is based on the product of the basic creep coefficient depending on the age of the concrete and function of the creep in time. The product form is used in several national codes as ACI 318-08 and EN1992-1-1,

$$\varphi(\mathrm{t},\tau) = \varphi_0(\tau)\left[\frac{t-\tau}{\beta_{\mathrm{H,t}}+t-\tau}\right]. \tag{7.28}$$

As first, transversal spreading of the traffic load systems has been done for determining of bending moment maximum in each girder. The results are mentioned in the following table. According to the load distribution, the particular load systems were modified.

Tab. 7.6 Transversal spreading of the load

| Beam | 1 | 2 | Sum |
|---|---|---|---|
| Span | 3195 kNm | 1726 kNm | 4921 kNm |
| | 64.9 % | 35.1 % | 100 % |

### 7.3.1 Objective function

Generally, the post-tensioning consists of eight tendons in one girder. The tendons with geometry D have the same geometry. Finally, four different tendon geometries are optimized. As has been mentioned in Chapter 7.1.3, the objective function usually includes a total mass of tendons. Nevertheless, we optimize also dimensions of the girder. Therefore, it is more efficient to introduce objective function based on the total cost of the structure. This includes cost for prestressing reinforcement and the cost for concrete,

$$f(\mathrm{x}) = \min(C_{\mathrm{m}}) = C_{\mathrm{p}} \cdot \left(\sum_{\mathrm{i}=1}^{\mathrm{n}} A_{\mathrm{p,i}} \cdot n_{\mathrm{t,i}} \cdot n_{\mathrm{g,i}} \cdot L_{\mathrm{p,i}} \cdot \gamma_{\mathrm{p,i}}\right) + C_{\mathrm{c}} \cdot A_{\mathrm{c}} \cdot L_{\mathrm{tot}}. \quad (7.29)$$

where $A_{\mathrm{p,i}}$ is the area of one strands Y1770S7-15.7; values $n_{\mathrm{t,i}}$ and $n_{\mathrm{g,i}}$ are a numbers of strands in tendon $i$ and a number of the same tendons, respectively; $L_{\mathrm{p,i}}$ is a length of the particular tendon; $\gamma_{\mathrm{p,i}}$ is an unity mass of the prestressing tendons; $A_{\mathrm{c}}$ is a area of concrete cross-section; $L_{\mathrm{tot}}$ is a total length of the bridge; $C_{\mathrm{p}}$ and $C_{\mathrm{c}}$are the cost for concrete and prestressing steel unit defined as $C_{\mathrm{p}} = 4.0$ €/kg and $C_{\mathrm{c}} = 216.0$ €/m$^3$, respectively; $i = 1 \dots n$ where $n$ is a number of particular tendon geometry.

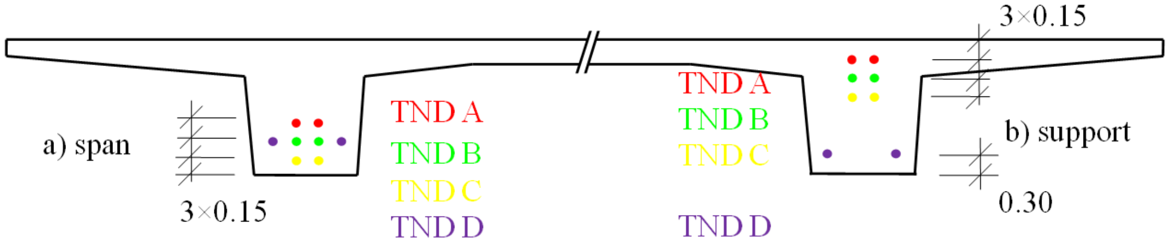



Fig. 7.20 Distribution of the tendons in the cross-section –a) span, b) support

### 7.3.2 Design variables

The structure was parameterized similarly as described in 7.1.2. Totally 11 independent variables were used. The parameters are used for the vertical and horizontal geometry of the tendons and for the number of the strands. Three layers of post-tensioning are applied (see Fig. 7.21). A constant axial distance (150 mm) between them is fixed in the span and above the support. The distribution of the tendons in the cross-section is visible in Fig. 7.20. Additionally, the vertical ($H_1$) and horizontal ($B_1$) dimensions of the girder were optimized (see Fig. 7.18). There are also other parameters depending on the independent variables.

Tab. 7.7 Initials and limits for design variables in the 2nd span

| Parameter | $n_{tABC}$ [-] | $n_{tD}$ [-] | $n_{gD}$ [-] | $x_{A6}$ [m] | $x_{A10}$ [m] | $z_{A2}$ [m] | $z_{A4}$ [m] | $z_{B2}$ [m] | $z_{B4}$ [m] | $B_1$ [m] | $H_1$ [m] |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Initial | 15 | 15 | 2 | 5.00 | 5.00 | 0.80 | 1.00 | 0.60 | 0.80 | 1.20 | 1.10 |
| Min. | 11 | 11 | 0 | 3.50 | 3.50 | 0.45 | 0.45 | 0.45 | 0.45 | 1.10 | 1.10 |
| Max. | 15 | 17 | 2 | 8.00 | 8.00 | 1.10 | 1.10 | 0.95 | 0.95 | 1.40 | 1.40 |
| Step | 2 | 2 | 1 | 0.25 | 0.25 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |

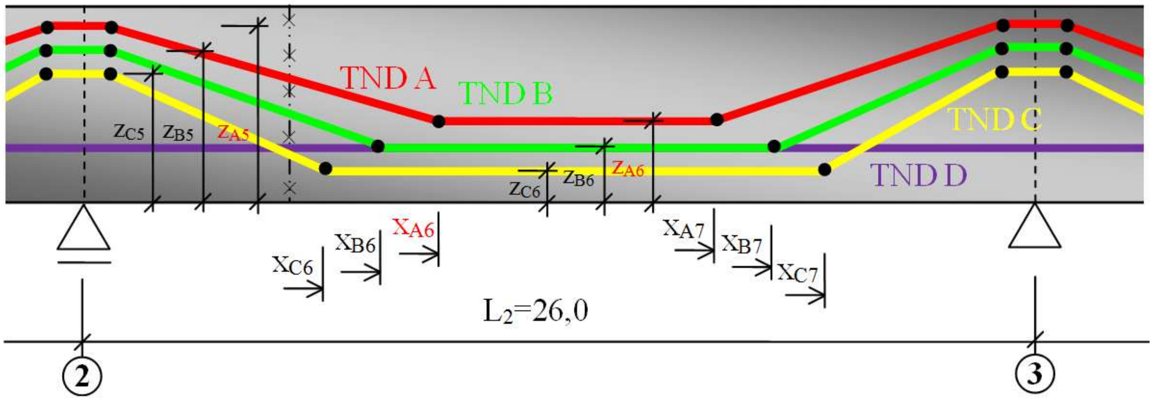



Fig. 7.21 Parameterization of tendons in the 2nd span

### 7.3.3 Constraints

The structure has been optimized based on several criteria according to Eurocodes. Generally, the first (SLS) and the second (ULS) limit states were verified. The allowable concrete stresses were checked in each construction stages when the particular span was cast and prestressed. An ultimate cross-section capacity is checked using interaction diagram. Limit values of check (ratio of calculated and limit value) was selected in standard way 1.0. Used constraint can be divided into the following groups.

- Geometrical – these constraints are coming from geometry of cross-section and distribution of the individual tendon in the cross-section with respect of minimal concrete cover and clear distances between tendons.
- Allowable concrete stresses – normal stresses were checked for each construction stage within service traffic loads. The maximal and minimal stresses in concrete are compared with allowable concrete stress in tension $\left(f_{\text{ct,eff}} = 3.76\text{ MPa}\right)$ and in compression $\left(\sigma_{\text{cc,ch}} = 0.6\ f_{\text{ck}} = 21\text{ MPa}\right)$.

- *Construction*
  - Allowable compressive stress – normal stresses on characteristic combination during construction are evaluated as the indication of the longitudinal cracks existence (see formula 7.2 (2) from EN1992-1-1) as

$$g_1(x) = \frac{\sigma_{\mathrm{cc}}}{\sigma_{\mathrm{cc,ch}}} - 1 \leq 0 \,, \tag{7.30}$$

    where $\sigma_{\mathrm{cc,ch}} = 0.6 \cdot \mathrm{f_{ck}} = 21$ MPa.

  - Allowable tensile stress – normal stresses on quasi-permanent combination during construction are compared with maximal tensile strength according to Chapter 113.3.2 (103) from EN1992-2 as follows

$$g_2(x) = \frac{\sigma_{\mathrm{ct}}}{k \cdot f_{\mathrm{ctm}}(\mathrm{t})} - 1 \leq 0. \tag{7.31}$$

- *Service*
  - Allowable compressive stress – normal stresses on characteristic combination during service in 100 years are evaluated similarly as during construction stages as

$$g_3(x) = \frac{\sigma_{\mathrm{cc}}}{\sigma_{\mathrm{cc,ch}}} - 1 \leq 0 \,. \tag{7.32}$$

  - Allowable tensile stress – normal stresses on characteristic combination during service in 100 years are compared with maximal tensile strength $\left(f_{\mathrm{ct,eff}} = 3.76\ \mathrm{MPa}\right)$ according to Chapter 7.1(2) from EN1992-1-1 as follows

$$g_4(x) = \frac{\sigma_{\mathrm{ct}}}{f_{\mathrm{ct,eff}}} - 1 \leq 0 \,. \tag{7.33}$$

- Capacity in ULS – the verification of cross-section in ULS is performed using interaction diagram (N+M loads) and shear check (Vz).
  - *Interaction diagram* - verification of beam loaded by combination of normal force and bending moment provides an interaction diagram. A fundamental STR/GEO Set B combination is used for verification in this check based on

$$g_5(x) = \frac{N_{\mathrm{Ed}}}{N_{\mathrm{u}}} - 1 \leq 0 \,, \tag{7.34}$$

$$g_6(x) = \frac{M_{\mathrm{Ed}}}{M_{\mathrm{u}}} - 1 \leq 0 \,. \tag{7.35}$$

- *Shear check* – an acting shear forces is compared with shear capacity of concrete cross-section. The minimal required shear reinforcement is considered only. The constraint is expressed as

$$g_7(\mathrm{x}) = \frac{V_{\mathrm{Ed}}}{V_{\mathrm{Rd,c}}} - 1 \leq 0\ . \tag{7.36}$$

### 7.3.4 Optimization algorithm and results

As was used for the previous examples, the evolutionary genetic algorithm is suitable for this case. When number of strand is discrete design variables, the method enabling optimization of them is recommended. A performed testing proved the method of modified simulated annealing as suitable for this example. This method allows to set changing step of design variables. The population consisted of 37 members was used. The initial annealing temperature was calculated based on acceptance of 50 % of members from the first population is equal to $T_{\max} = 13720.2$. The annealing constant was $T_{\mathrm{mult}} = 0.661$ in selected amount of 10 iterations.

Totally, 237 iterations have been done in 76hrs and 56minutes. A construction stages calculation within time dependent analysis was performed for each iteration. A dependency of objective function on iteration steps shows Fig. 7.22. Note that all solutions are presented in this figure, i.e. including solutions that do not fulfil given constraints.

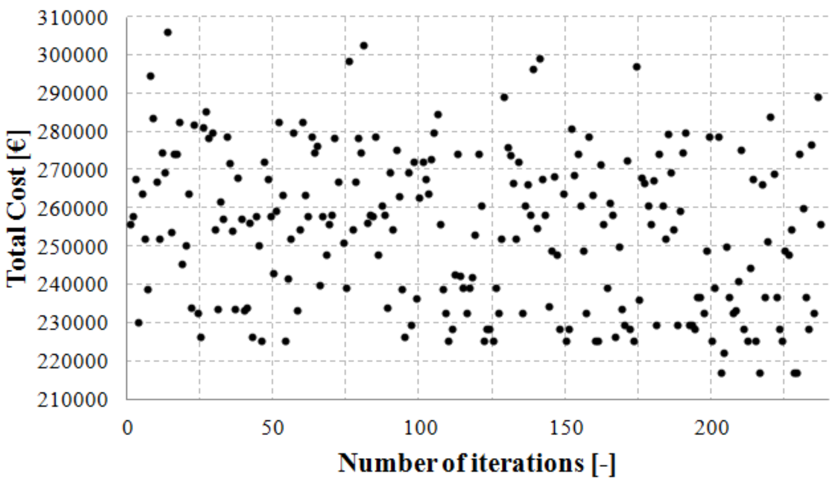



Fig. 7.22 Evaluation of objective function on number of iterations

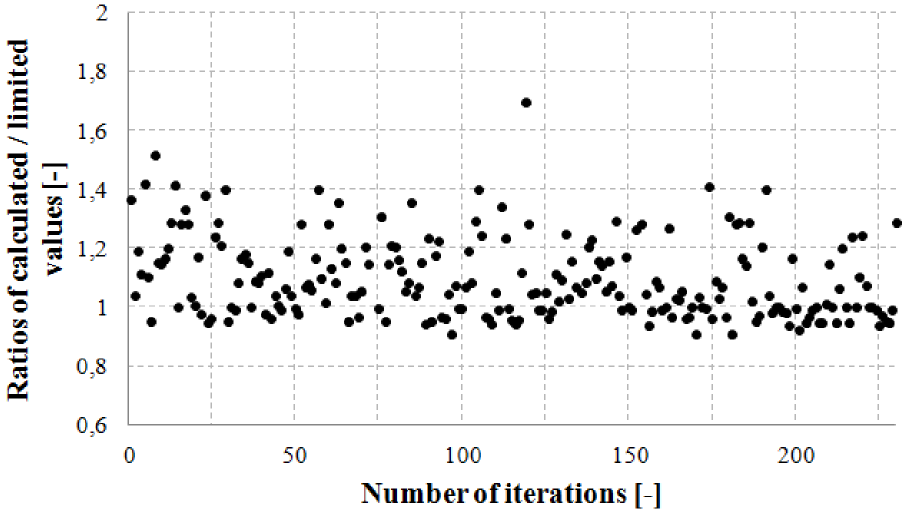



Fig. 7.23 Evaluation of constraints on number of iterations

If we compare the maximal ratios of calculated / limited values from all used checks in Fig. 7.23, there is clearly visible convergence to solutions with ratio ≤ 1.0. The solutions with ratio > 1.0 are automatically refused.

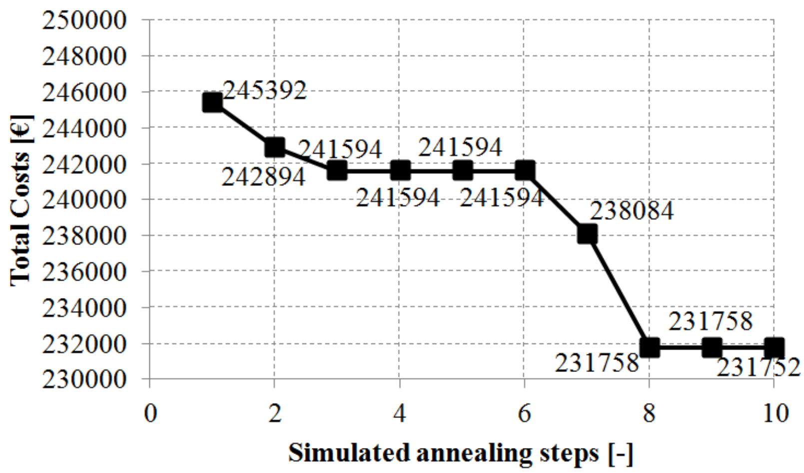



Fig. 7.24 Evaluation of tendon mass on number of iterations

The modified simulated annealing method found several optimum in particular annealing steps. The Fig. 7.24 shows the values of objective function depending on annealing steps. After deeper verification of the optimal solutions, we selected four representatives mentioned in Tab. 7.8. The total cost of the structure significantly increases during the annealing. The found optimum (solution 1 in Tab. 7.8) characterizes objective function 231 752 €. The optimal shape of girder is 1.1 × 1.1 m. It concludes to decreasing of girder width by 10 cm. When the 11 strands in one tendon of geometry A can be used, the required number of strands is reduced by 4. Additionally, the tendon with geometry D is not necessary to use at all.

Tab. 7.8 Comparison of optimum with initials

| Parameter / Solution | $n_{t,ABC}$ [-] | $n_{t,D}$ [-] | $n_{g,D}$ [-] | $x_{A6}$ [m] | $x_{A10}$ [m] | $z_{A2}$ [m] | $z_{A4}$ [m] | $z_{B2}$ [m] | $z_{B4}$ [m] | $B_1$ [m] | $H_1$ [m] |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Initial | 15 | 15 | 2 | 5.00 | 5.00 | 0.80 | 1.00 | 0.60 | 0.80 | 1.20 | 1.10 |
| Sol. 1 | 11 | 17 | 0 | 4.75 | 3.75 | 1.10 | 0.80 | 0.90 | 0.65 | 1.10 | 1.10 |
| Sol. 2 | 11 | 17 | 0 | 4.75 | 5.00 | 0.50 | 0.80 | 0.85 | 0.65 | 1.10 | 1.10 |
| Sol. 3 | 11 | 17 | 0 | 4.75 | 4.25 | 0.50 | 0.45 | 0.65 | 0.80 | 1.15 | 1.15 |
| Sol. 4 | 11 | 17 | 0 | 4.25 | 3.50 | 0.85 | 0.95 | 0.70 | 0.75 | 1.15 | 1.20 |

As you can see from Fig. 7.25 and Fig. 7.26, the maximal ratios of cross-section resistance check in ULS and allowable concrete stresses check are very close to limit 1.0. A tensile stress 3.75 MPa appears for the characteristic combinations which is still less than mean tensile strength of concrete 3.76 MPa and cracks do not appear. This constraint was detected as the most dangerous during optimization. Maximal calculated ratios are presented in table Tab. 7.9.

Tab. 7.9 Ratios of particular checks for optimums

| Solution | Check A [-] | Check B [-] | Check C [-] | Total Cost [€] | Save [%] |
|---|---|---|---|---|---|
| Initial | 0.86 | 0.79 | 0.37 | 277 802 | - |
| Sol. 1 | 1.00 | 0.91 | 0.41 | 231 752 | 16,6 |
| Sol. 2 | 0.94 | 0.70 | 0.42 | 231 758 | 16,6 |
| Sol. 3 | 0.97 | 0.88 | 0.41 | 238 084 | 14,3 |
| Sol. 4 | 0.99 | 0.87 | 0.43 | 241 594 | 13,0 |

Where Check A – check of allowable concrete stresses; maximum from the construction and serviceability stages; Check B – check of capacity of cross-section using interaction diagram in ULS; Check C – check of shear capacity.

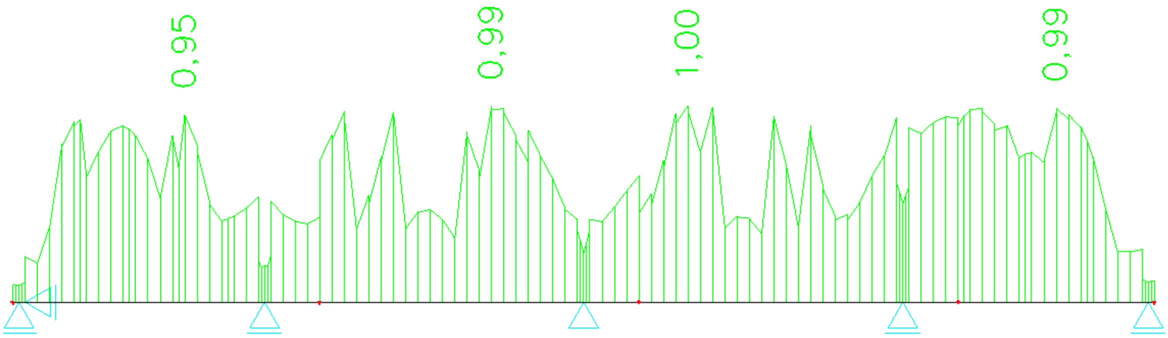



Fig. 7.25 Ratio of allowable concrete stresses check on characteristic combination in 100 years

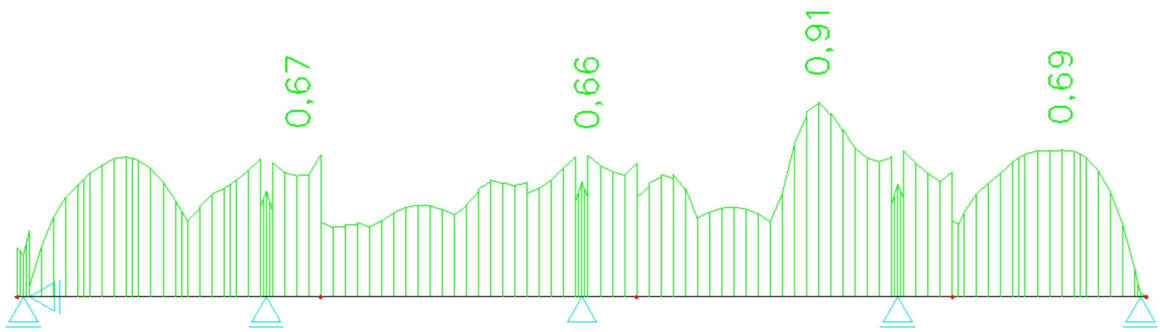



Fig. 7.26 Ratio of ULS check (interaction diagram) in 100 years

Fig. 7.27 presents the comparison of the initial and optimized tendon geometries. The main differences are in the first and fourth span. Nevertheless, the first span is only drawn to be differences more visible. We can see that tendons A and B significantly change its geometry and became more effective. Note, the figure is 5 times vertically scaled.

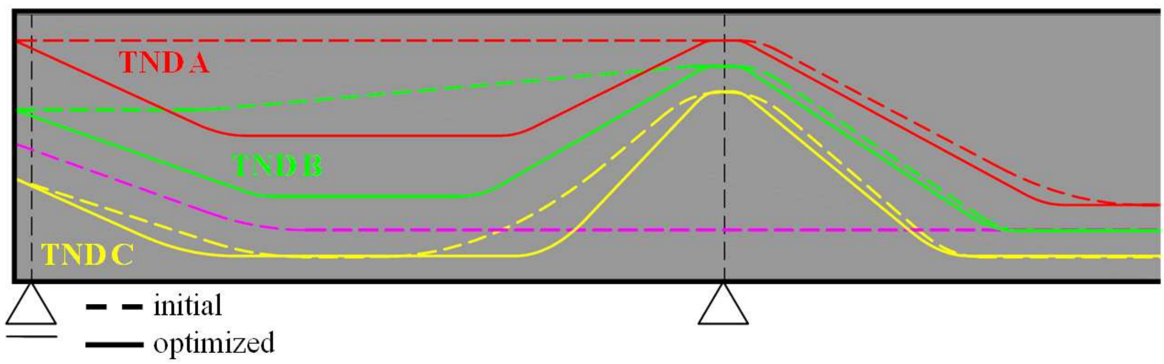



Fig. 7.27 Comparison of tendon geometries

## 7.4 Nine span post-tensioned deck bridge

The last example of the post-tensioned bridge optimization procedure is the nine spans bridge with deck cross-section. Here, the large structure with many construction stages is very complex example for optimization. As was stated, the bridge consists of nine spans (22.0 + 27.0 + 27.0 + 27.0 + 34.0 + 25.0 + 23.0 + + 23.0 + 19.0 m) with total length 227.0 m, see Fig. 7.29. A span-by-span technique with overhang cantilever is applied in the construction process. A relatively thick cross-section from C30/37 is drawn in Fig. 7.28. Similarly like in case of double T-section bridge (see Chapter 7.3), the construction of each span includes two phases. At first, the concrete is casted-in-place to fixed formwork and next, the prestressing of the span is applied. This procedure repeats for each span and completely nine groups of the construction stages are modelled. The prestressing is applied incrementally across two spans. The couplers connect the half of the tendons in vertical construction joint of each cantilever overhang. At least 50 % of tendons have to go through the construction joint.

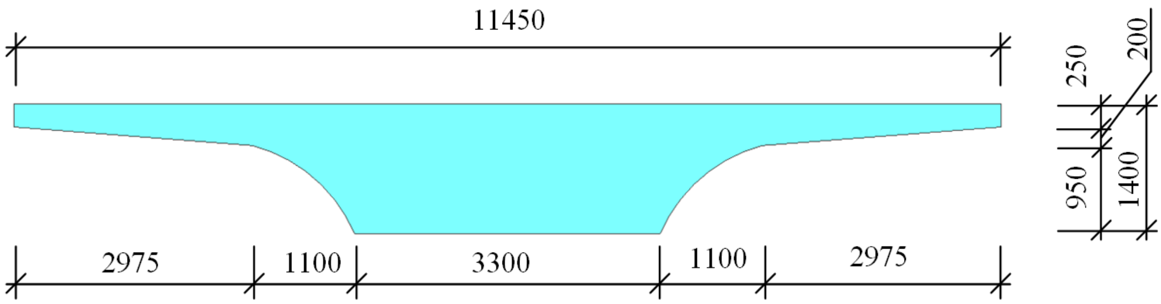



Fig. 7.28 Deck cross-section

The structure is analyzed using construction stages calculation taking into account an effect of span-by-span construction within the time dependent calculation [92] covering rheological behaviour of concrete creep and shrinkage according to EN1992-1-1. The process of construction starts in the fifth span, next, span no. 4 and 6 are symmetrically built. Afterwards, the construction continues with the right side of the bridge (spans no.7, 8 and 9). Finally, the construction of spans no. 3, 2 and 1 gets complete the bridge. The construction stages analysis automatically includes changes in static system of the structure. Overall scheme of construction is shown in Fig. 7.30.

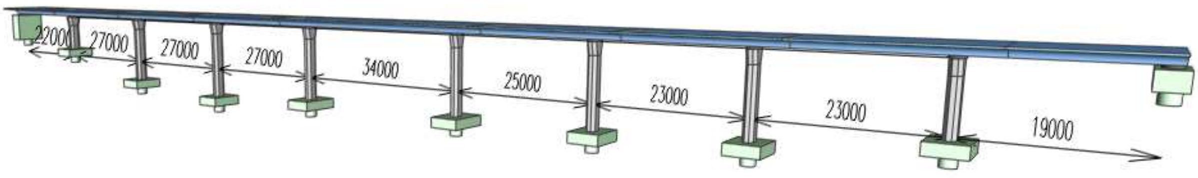



Fig. 7.29 Bridge structural overview

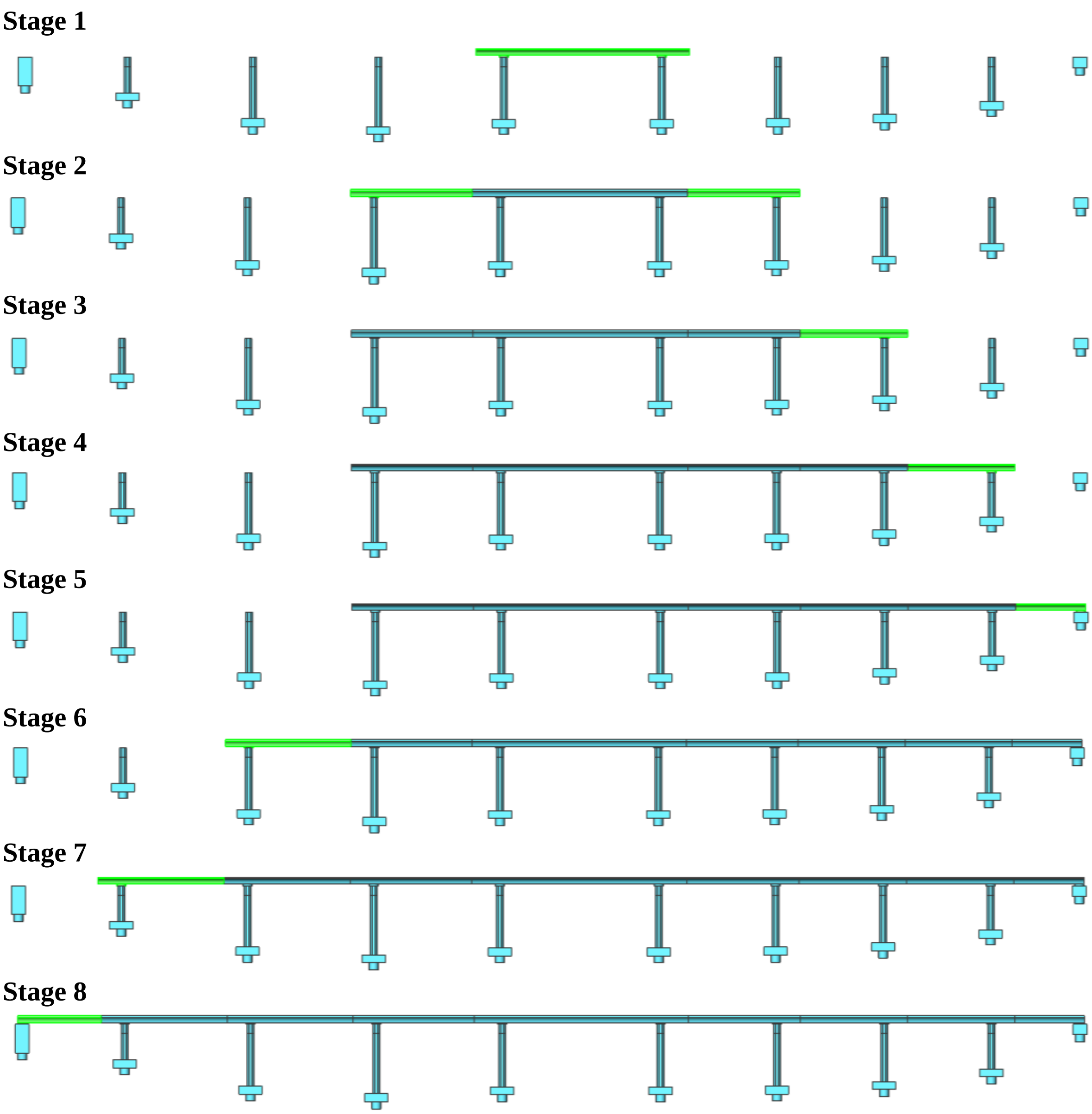



Fig. 7.30 Construction stages process

### 7.4.1 Objective function

Because the cross-section has the constant dimensions, the purpose of this study was to minimize a total mass of tendons ($M_p$) in the bridge with more effective tendon geometry. Fig. 7.31 shows the typical distribution of the tendons above the support and in the middle of the span. The bridge is prestressed using span-by-span technology. At first the half of the tendons is applied in the span. During the construction of the neighbouring span, two adjacent spans are prestressed by the continuous tendon over them. As was mentioned above, the prestressing of bridge consists of two different geometries (A, B) for each span. Additionally, the prestressing is extended by tendons with geometry C in the span 5. An objective function can be expressed as follows

$$f(\mathrm{x}) = \min\left(M_{\mathrm{p}}\right) = \sum_{\mathrm{i}=1}^{\mathrm{n}} A_{\mathrm{p,i}} \cdot n_{\mathrm{t,i}} \cdot n_{\mathrm{g,i}} \cdot L_{\mathrm{p,i}} \cdot \gamma_{\mathrm{p,i}}, \qquad (7.37)$$

where $A_{\mathrm{p,i}}$ is the area of one strands Y1770S7-15.7; values $n_{\mathrm{t,i}}$ and $n_{\mathrm{g,i}}$ are a numbers of strands in tendon $i$ and a number of the same tendons, respectively; $\mathrm{L}_{\mathrm{p,i}}$ is a length of the particular tendon; $\gamma_{\mathrm{p,i}}$ is an unity mass of the prestressing tendons; $i = 1 \ldots n$ where $n$ is a number of particular tendon geometry.

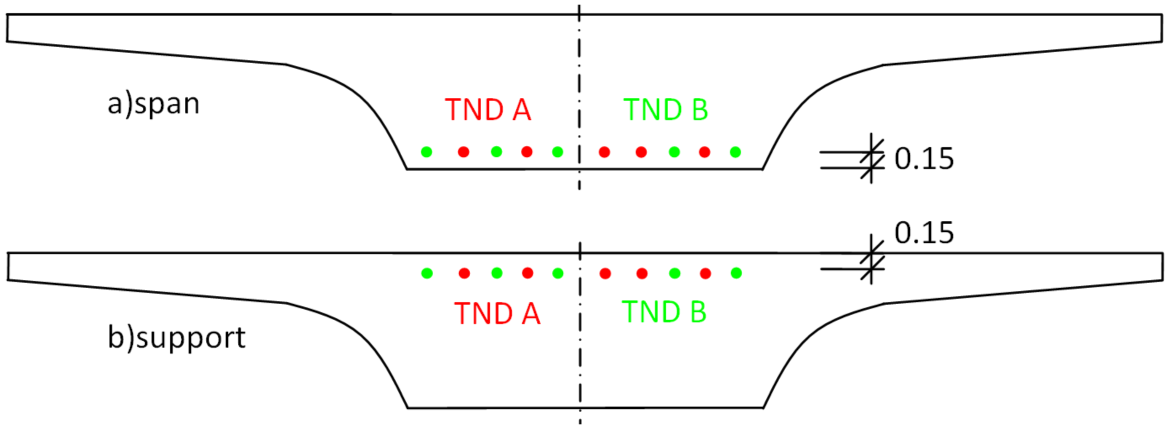



Fig. 7.31 Distribution of the tendons in the cross-section – a) span, b) support

### 7.4.2 Design variables

Totally 24 independent variables were used. The parameters are used for the number of the strands (see Tab. 7.9), horizontal geometry of the tendons and initial stress during prestressing (see Tab. 7.11). A typical span with post-tensioned tendons is shown in Fig. 7.32.

Tab. 7.10 Initials, range and step limits of design variables for number of strands

| Parameter | $n_{\mathrm{tA}}$ [-] | $n_{\mathrm{tB}}$ [-] | $n_{\mathrm{tC}}$ [-] | $n_{\mathrm{gA}}$ [-] | $n_{\mathrm{gB}}$ [-] | $n_{\mathrm{gC}}$ [-] |
|---|---|---|---|---|---|---|
| Initial | 19 | 19 | 19 | 5 | 5 | 5 |
| Min. | 15 | 15 | 15 | 2 | 2 | 2 |
| Max. | 19 | 19 | 19 | 5 | 5 | 5 |
| Step | 2 | 2 | 2 | 1 | 1 | 1 |

Tab. 7.11 Initials, range and step of design variables for tendon geometry

| Parameter | Initial | Min. | Max. | Step | Optimum sol. no.1 |
|---|---|---|---|---|---|
| $x_{2B03}$ [m] | 13.5 | 10.3 | 13.9 | 0.1 | 13.3 |
| $x_{3A04}$ [m] | 9.2 | 9.2 | 10.4 | 0.1 | 9.4 |
| $x_{3A08}$ [m] | 44.0 | 36.2 | 45.9 | 0.1 | 36.3 |
| $x_{3A12}$ [m] | 74.0 | 70.2 | 75.4 | 0.1 | 74.4 |
| $x_{4B03}$ [m] | 12.6 | 12.6 | 14.8 | 0.1 | 12.6 |
| $x_{4B06}$ [m] | 26.0 | 23.2 | 27.4 | 0.1 | 25.7 |
| $x_{5A03}$ [m] | 12.1 | 12.1 | 13.8 | 0.1 | 13.0 |
| $x_{5A06}$ [m] | 24.5 | 22.2 | 26.4 | 0.1 | 25.5 |
| $x_{6B03}$ [m] | 12.1 | 12.1 | 13.6 | 0.1 | 12.2 |
| $x_{6B06}$ [m] | 23.0 | 23.0 | 24.5 | 0.1 | 23.5 |
| $x_{7A04}$ [m] | 12.0 | 9.2 | 15.4 | 0.1 | 12.8 |
| $x_{7A08}$ [m] | 36.2 | 36.2 | 39.5 | 0.1 | 36.7 |
| $x_{8B04}$ [m] | 12.0 | 9.2 | 15.4 | 0.1 | 11.4 |
| $x_{8B08}$ [m] | 36.2 | 36.2 | 40.0 | 0.1 | 36.2 |
| $x_{9A02}$ [m] | 5.5 | 4.6 | 6.0 | 0.1 | 5.1 |
| $x_{9A06}$ [m] | 27.2 | 27.2 | 31.0 | 0.1 | 29.6 |
| $x_{2C04}$ [m] | 18.0 | 10.3 | 19.9 | 0.1 | 12.8 |
| $x_{6C03}$ [m] | 10.6 | 10.6 | 11.9 | 0.1 | 11.0 |
| $x_{9C02}$ [m] | 5.5 | 3.1 | 5.9 | 0.1 | 4.6 |
| $\sigma_p$ [MPa] | 1400 | 1350 | 1400 | 5.0 | 1360 |

Simultaneously, the tendon layout has been parameterized. Thanks to active role of prestressing, we can change distribution of internal forces using a modification of tendon geometry. The independent variables of the tendon geometry are displayed in red colour in Fig. 7.32.

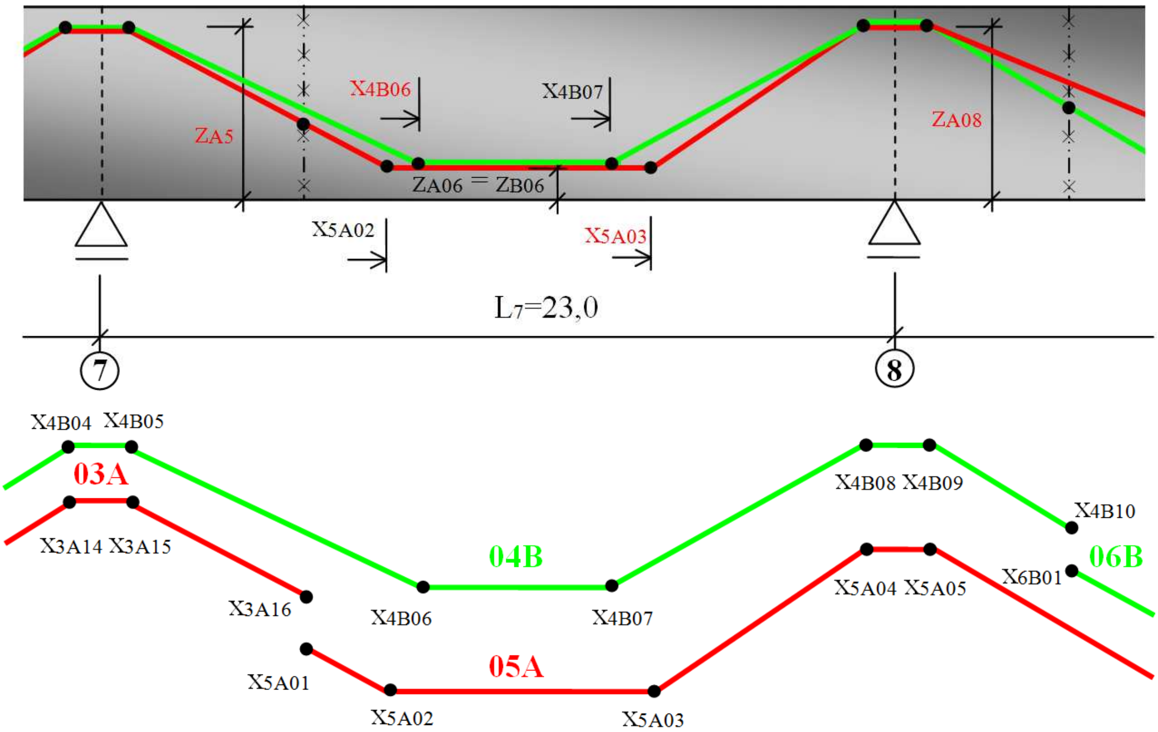



Fig. 7.32 Tendon scheme in typical span

### 7.4.3 Constraints

As has been discussed in chapter 7.1.4, the concrete bridges have to satisfy many kinds of design checks. Nevertheless, not all of them must be necessarily included in the optimization. Therefore, the check of allowable concrete stresses during construction and in services has been introduced as constraint in the optimization. An ultimate cross-section capacity is checked using interaction diagram. A shear effect is verified by comparing shear capacity of cross-section with acting shear force. Because the structure is relatively long and thick, fatigue verification can be also significant. The fatigue check is performed for concrete and prestressing reinforcement separately, see Chapter 7.1.4. In general, the constraints are the same like for the previous example (see double T-section bridge in Chapter 7.3.3). Additionally, the fatigue check is performed according to EN1992-2 (see constraints for fatigue verification mentioned in 7.1.4).

### 7.4.4 Optimization algorithm and results

This large structure is built and prestressed in many construction stages. Thus, to determine optimum geometry and number of tendons cannot be simple procedure. In order to make correct optimization, the method of modified simulated annealing has been used. This algorithm has found several optimums from the population of 50 members. These optimums were analyzed and only some of them satisfied all of the checks required by the code. Totally, 414 iterations have been done. Unfortunately, the whole optimization procedure last 274 hrs. The most of not accepted solutions were refused due to not satisfying check of allowable concrete stresses for characteristic combination. The calculated concrete stresses very often exceeded allowable tensile strength of concrete during execution. The comparison of the accepted optimum with the initial state is illustrated in Tab. 7.12 and Tab. 7.13. The solution no.1 allows like for initial solution the using of 19 strand tendons. Nevertheless, it is possible to use only four tendons of geometry A and B. In addition, the tendon with geometry C consists of only 17 tendons. The optimal values of tendon geometry are shown in Tab. 7.11 and number of strands and tendons in Tab. 7.11. This configuration of optimum solution brings saving about 18%. Next, the optimization offers also other solutions. However, the solution save huge amount of kilograms of post-tensioned reinforcement, they cannot be used due to not satisfying of constraints (see Tab. 7.12).

Tab. 7.12 Comparison of optimum with initials

| Parameter | $n_{tA}$ [-] | $n_{tB}$ [-] | $n_{tC}$ [-] | $n_{gA}$ [-] | $n_{gB}$ [-] | $n_{gC}$ [-] |
|---|---|---|---|---|---|---|
| Initial | 19 | 19 | 19 | 5 | 5 | 5 |
| Sol. 1 | 19 | 19 | 17 | 4 | 4 | 5 |
| Sol. 2 | 17 | 17 | 15 | 4 | 4 | 5 |
| Sol. 3 | 15 | 15 | 19 | 5 | 5 | 3 |
| Sol. 4 | 19 | 19 | 17 | 5 | 5 | 5 |

Tab. 7.13 Ratios of particular check for optimums

| Solution | Check A [-] | Check B [-] | Check C [-] | Check D [-] | Check E [-] | $M_p$ [kg] | Save [%] |
|---|---|---|---|---|---|---|---|
| Initial | 0.921 | 0.821 | 0.913 | 0.772 | 0.965 | 56565.45 | - |
| Sol. 1 | 1.000 | 0.911 | 0.991 | 0.789 | 0.978 | 46099.99 | 18.5 |
| Sol. 2 | 0.999 | 0.939 | 1.008 | 0.799 | 0.997 | 41153.04 | 27.2 |
| Sol. 3 | 0.987 | 0.902 | 1.011 | 0.796 | 0.996 | 42997.48 | 23.9 |
| Sol. 4 | 0.941 | 0.840 | 0.969 | 0.788 | 0.982 | 55640.56 | 1.6 |

Where CheckA – check of allowable concrete stresses; maximum from the construction and serviceability stages; CheckB – check of capacity of cross-section using interaction diagram in ULS; CheckC – check of shear capacity; CheckD – maximum ratio of fatigue check for concrete and prestressing reinforcement; CheckE – check of prestressing reinforcement.

When we look on the distribution of the maximal ratio from all checks on the number of iterations (see Fig. 7.33), we can conclude this example is highly constrained. There is visible tendency to converge close to 1.0 but only several solutions fulfil the constraints of checks. Furthermore, this effect is also transparent form the dependency of objective function on number of iterations (see Fig. 7.34). The MSA algorithm hardly finds the optimum and thus, the value of objective functions is spread over the whole investigated space.

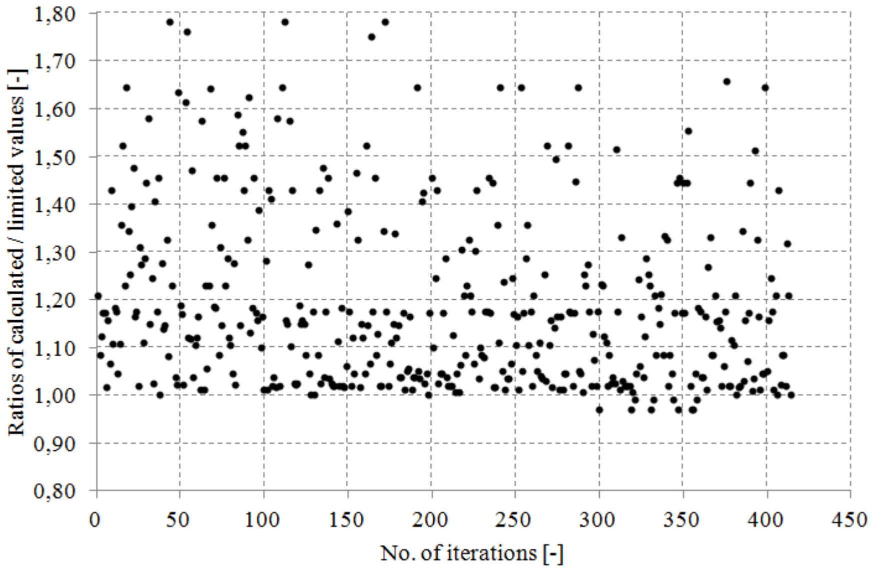



Fig. 7.33 Evaluation of maximal constraints on number of iterations

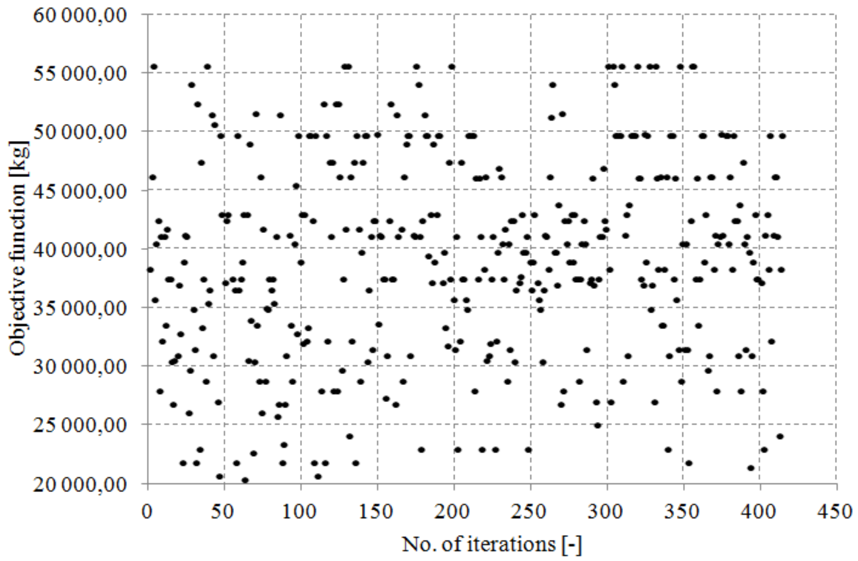



Fig. 7.34 Evaluation of objective function on number of iterations

The ratios of allowable concrete stresses check (Fig. 7.35), check of capacity using interaction diagram (Fig. 7.36) for solution no.1 are very close to limit ratio 1.0. These values are envelopes from all construction stages.

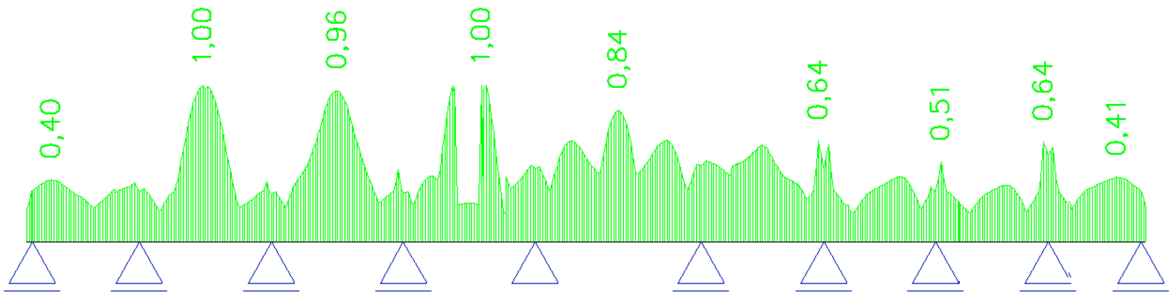



Fig. 7.35 Ratio of envelopes from all construction and serviceability stages for allowable concrete stresses check

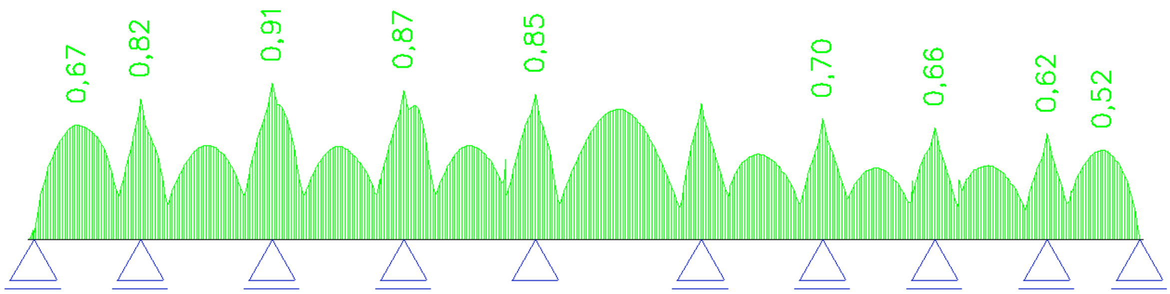
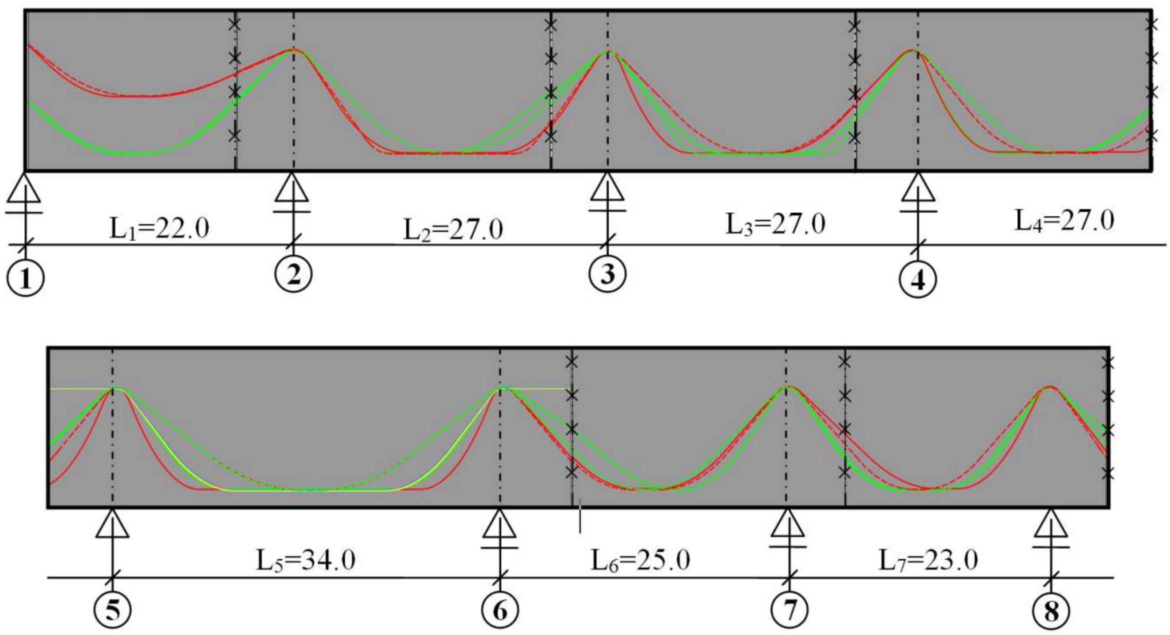



Fig. 7.36 Ratios of ULS by inter. diagram check for serviceability stage

If we compare tendon geometries for nine spans bridge, there are main differences in positions of horizontal arc vertexes. A full line represents optimal tendon geometry in comparison with initial dashed line.



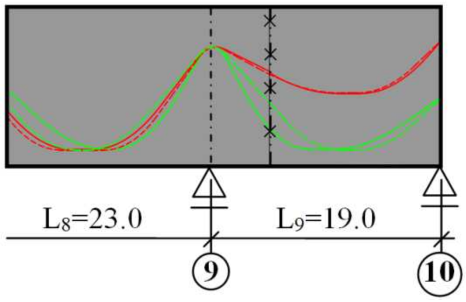



Fig. 7.37 Comparison of tendon geometries (dashed=initial; full=optimized)

## 7.5 Conclusions

At the beginning of this chapter, the preconditions for successful optimization of post-tensioned concrete bridges have been defined. The process of optimization was explained in particular chapters focused on parameterization, constraints and the selection of the optimization method. Special attention was paid to different types of objective function. In the previous examples we analyzed three typical cases of the optimization of post-tensioned concrete bridges. The bridges have been described and parameterized. The constraints were defined. A different shape of objective function has been used for a particular bridge. Finally, the comparison with the initial solution has been shown.

Consequently, the check of allowable concrete stresses has turned out to be the most critical check of the post-tensioned concrete bridge. To avoid an excessively time consuming calculation with many checks, it can be efficient to optimize the structure only for allowable concrete stresses.

In general, the results of the optimization process showed that the MSA method is a robust tool for optimization of such kind of structures. As can be seen for the optimization of a post-tensioned bridge, the rational saving depends on the defined type of objective function. The optimization processes were able decreasing of objective function about 24%, 17% and 18.5%, for three girders, double T-section and deck bridge, respectively.

This optimization process can be efficient for regular design of concrete bridges. Additionally, it can be suitable mainly in cases where the design of tendon geometry is affected by a complicated construction process. A typical case can be the design of tendon geometries in a vertical construction joint.

# 8 OPTIMAL DESIGN OF PRESTRESSED FLOOR SLAB BASED ON MINIMUM DEFLECTION

In previous chapters the optimization methods have been used for the design of prestressed concrete bridges. A typical suitable using of optimization algorithm for design of tendon geometry in floor slab is described in this chapter. The structure of a shopping centre designed by HELIKA a.s. consists of several blocks. The investigated part is situated in the second storey of one of them. Fig. 8.1 shows overall view on the block of the shopping centre. An application of hidden prestressed girder minimizes the using of internal column supporting the floor in the entrance hall. The outcome of this optimisation is to find such geometry of the hidden girder, geometry of the tendon and number of necessary strands to get minimal deflection of the slab.

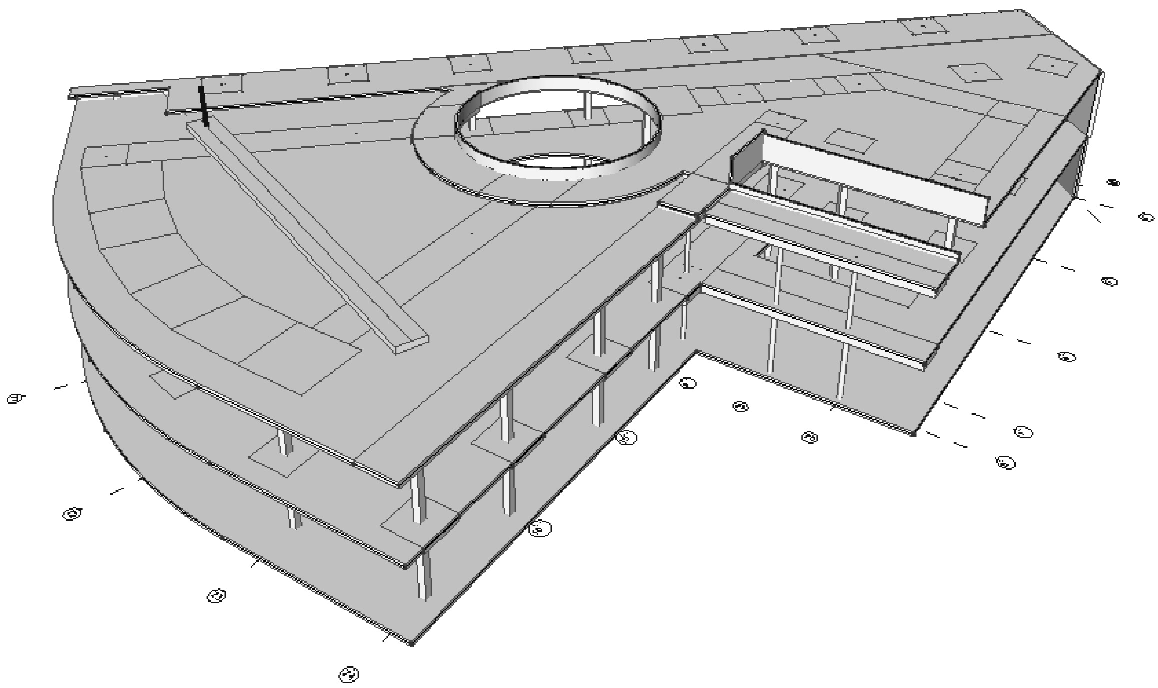


Fig. 8.1 Overall view on the block of the shopping centre

The circular and rectangular columns are made from concrete class C45/55 while the 350 mm depth floor slabs are from the C25/30. The permanent (selfweight, cast-in-place toping and dead) together with variable (commercial objects, escalator and fire truck) loads act on top storey. An analysis, design and checks are performed according to Eurocodes. A purpose of this example focuses on the top floor slab with hidden prestressed girder where the deformations from the dead and variable load are unfavourable.

The linear deflections from quasi-permanent combination (without prestressing) concentrate in the middle part of the slab (see Fig. 8.2). The maximum value is −12.6 mm. Therefore, designed hidden prestressed girder eliminates undesirable effects of deformation to acceptable values. It transversally transmits the deformation using prestressing tendons and spreads the load to the lateral members (see Fig. 8.3).

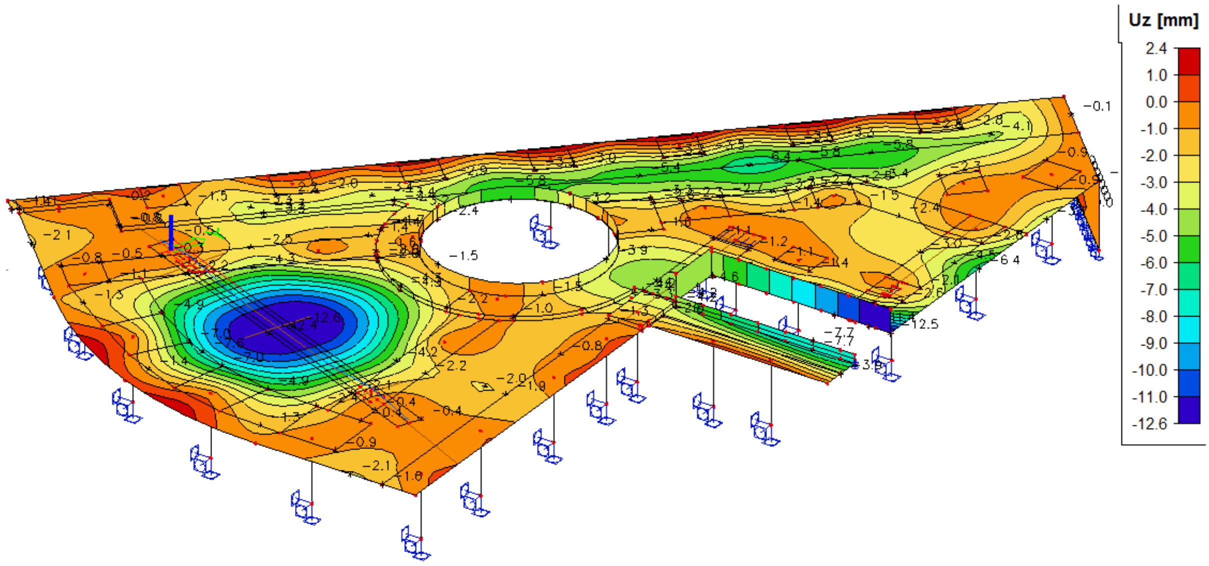



Fig. 8.2 Linear deflection from quasi-perm. combination (without prestressing)

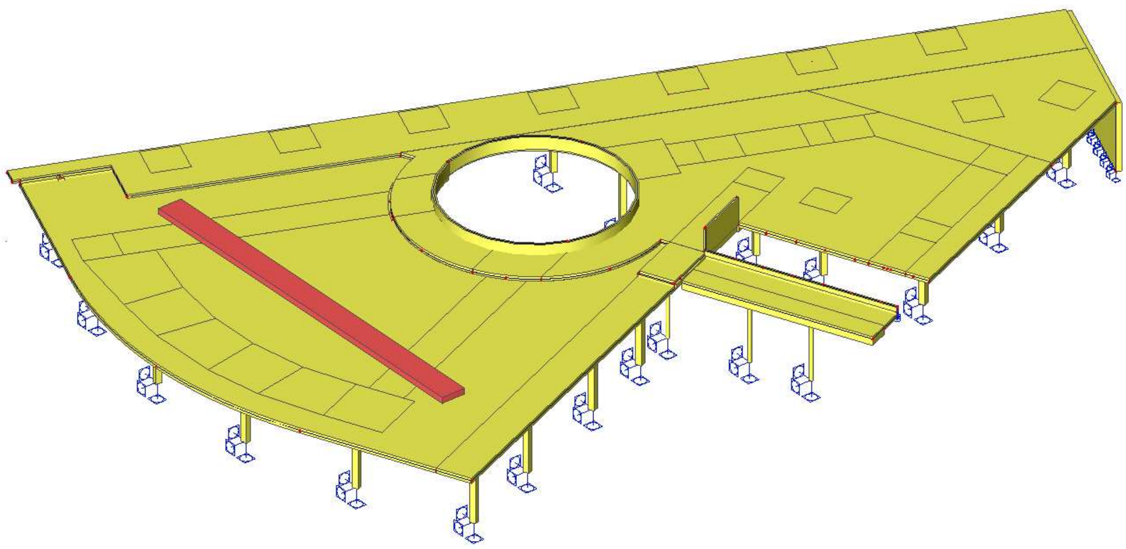


Fig. 8.3 Hidden girder (marked red) in the slab

## 8.1 Definition of optimization task and parameterization

The parameterization of hidden girder was the first step of the optimization. Both overhangs above the intermediate supports were selected as independent variables $L_1$ and $L_2$ (see Fig. 8.4).

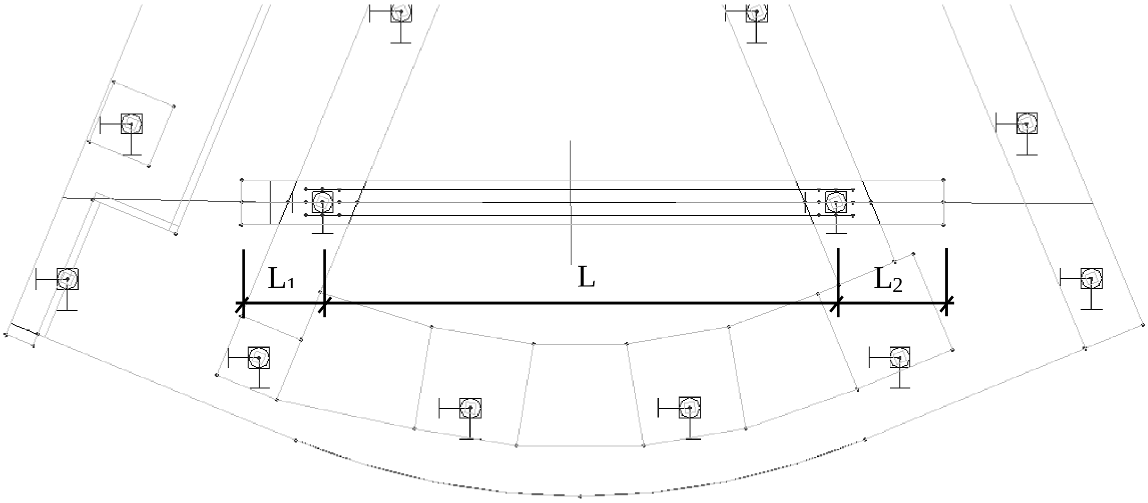



Fig. 8.4 Hidden girder parameters (view in XY plane)

A varying number of typical seven wires strands (Y1860S7-15.7) were prestressed against to rectangular hidden girder. Dependently calculated width $B$ (see formula 7.1) and a variable girder depth ($H$) are considered as cross-section dimensions. The last sets of variables were coordinates of tendon geometry (see Fig. 8.5). A maximal distance of tendon axis from the upper or lower surface remains unchanged with 90 mm. A vertical geometry of the tendon relates to bottom surface of the slab. The length and geometry of tendon were modified with respect of total length of the girder. Therefore, one independent variable Δz is only considered for calculation of vertical coordinate $z_1$ ($z_1 = z_2 - \Delta z$). Other parameters $z_2$ and $z_3$ are based on cross-section depth ($z_2 = H - c$) or constant ($z_3$ = const). The fixed properties of initial stresses (1430 MPa) with 6 mm anchorage set characterize used prestressing system. Finally, optimized numbers of strands were transformed to groups of “monostrands”. Each group consists of four of them (see Fig. 8.6). The total width of the girder ($B$) is calculated based on the number of required strands according to the following formula

$$B = 2 \cdot c_{\mathrm{nom}} + n_{\mathrm{group}} \cdot (w_{\mathrm{s}} + w_{\mathrm{d}}) + n_{\Delta} \cdot w_{\mathrm{s1}}, \tag{8.1}$$

where

$n_{\mathrm{t}}$ is a total number of required strands,

$n_{\mathrm{g1}}$ is a number of strands in group; default is 4,

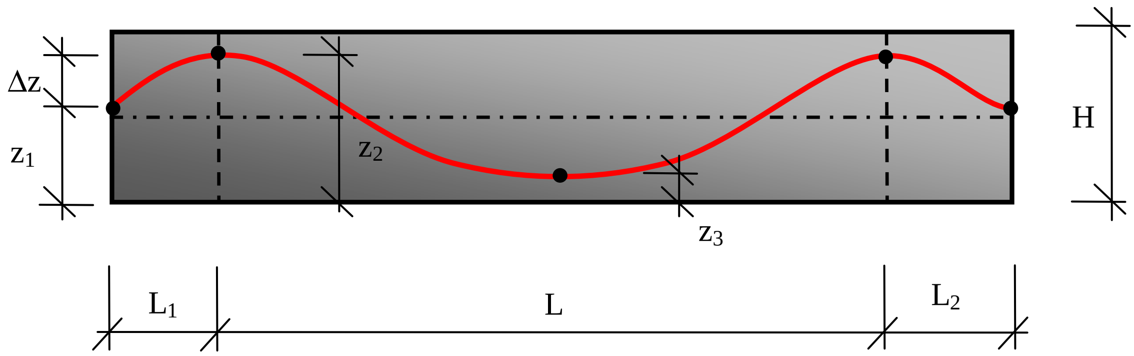



Fig. 8.5 Schematic tendon geometry (view in XZ plane)

$n_{\mathrm{group}}$ is a number of groups with $n_{\mathrm{g1}}$strands,

$$n_{\mathrm{group}} = \mathrm{roundup}\left(\frac{n_{\mathrm{t}}}{n_{\mathrm{g1}}} - 1\right), \tag{8.2}$$

$n_{\Delta}$ is a number of resting strands,

$$n_{\Delta} = n_{\mathrm{t}} - n_{\mathrm{g1}} \cdot n_{\mathrm{group}}, \tag{8.3}$$

$w_{\mathrm{s}}$ is a width of strand group with $n_{\mathrm{g1}}$ strands; $w_{\mathrm{s}} = n_{\mathrm{g1}} \cdot w_{\mathrm{s1}} = 0.08$ m,

$w_{\mathrm{d}}$ is a distance between group of strands; $w_{\mathrm{d}} = 0.08$ m,

$w_{\mathrm{s1}}$ is a width of one strands; $w_{\mathrm{s1}} = 0.02$ m,

$c_{\mathrm{nom}}$ is nominal cover of strands based on Chapter 4.4.1 from [133];$c_{\mathrm{nom}} =$ 0.04 m.

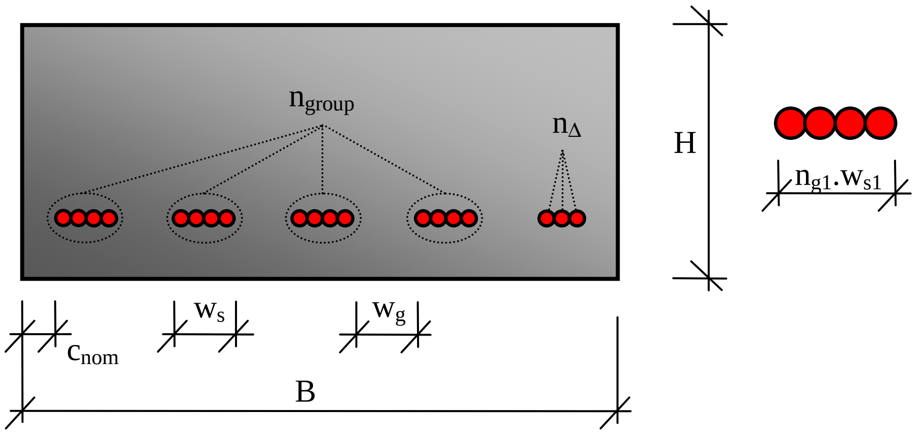



Fig. 8.6 Distribution of monostrands in the girder

Longterm losses of prestressing and rheological effect on concrete were neglected for this example. Tab. 8.1 shows all optimized parameters together with limits used in optimization algorithm.

The target of this optimisation is to find such geometry of the hidden girder, geometry of the tendon and number of necessary strands to get minimal linear deflection. In case of nonlinear deflection investigation, which is not part of this study, the results can be different due to changes of the axial force from prestressing. Hereby, the stiffness can be also changed and influences the results.

The presented study includes verification of minimal negative deflection in the span and positive above support of hidden girder. This example is complicated for design because each change of the overhang lengths significantly modifies deflection of the girder. Additionally, the overhangs influence the deflection outside of the girder parallel to longitudinal axis of the girder where upwards deflection concentrates too. This effect is clearly seen in the following figures with constant number of strands. A short overhangs (see Fig. 8.7) cause negative deflection in the span and positive outside of the overhangs. The increasing of the overhangs length (Fig. 8.9) decrease deflection outside of them and turn over deflection in the span to the positive values.

Tab. 8.1 Independent variables with limits

| Variable | Initial | Min | Max |
|---|---|---|---|
| $L_1$[m] | 3.138 | 2.0 | 4.8 |
| $L_2$[m] | 4.168 | 2.0 | 6.8 |
| $n_t$[-] | 55 | 40 | 60 |
| $\Delta z_1$[mm] | 300 | 50 | 300 |
| $H$ [m] | 0.75 | 0.35 | 0.75 |

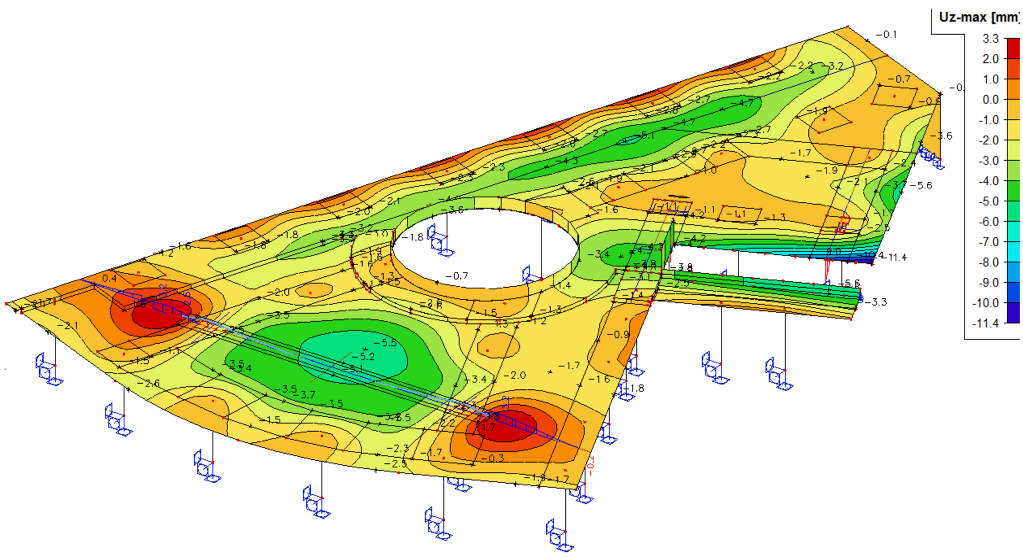



Fig. 8.7 Deflection with minimal lengths of overhangs ($L_1 = 2.0$ m,; $L_2 = 2.0$ m; minimum)

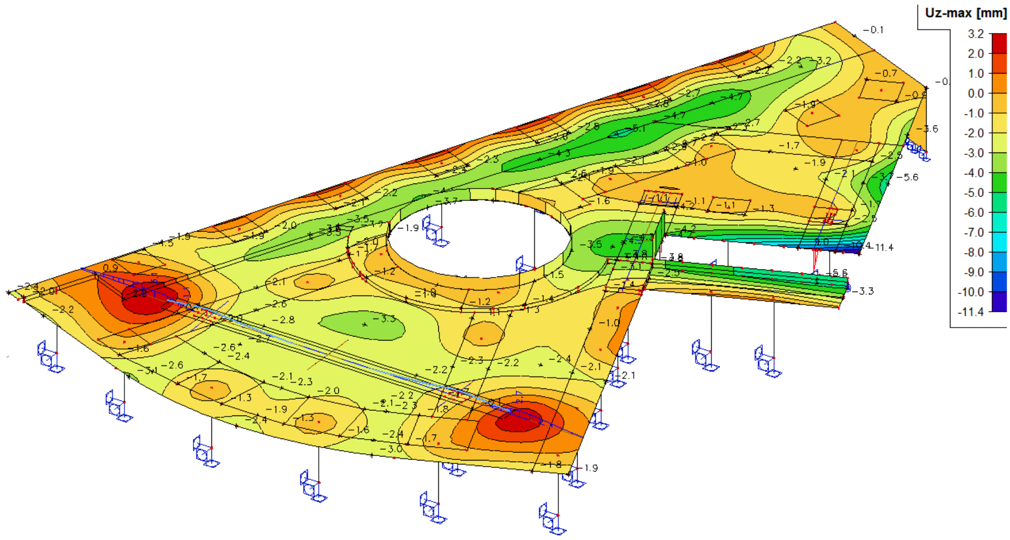



Fig. 8.8 Deflection with minimal lengths of overhangs ($L_1$ = 3.138 m,; $L_2$ = 4.168 m; initials)

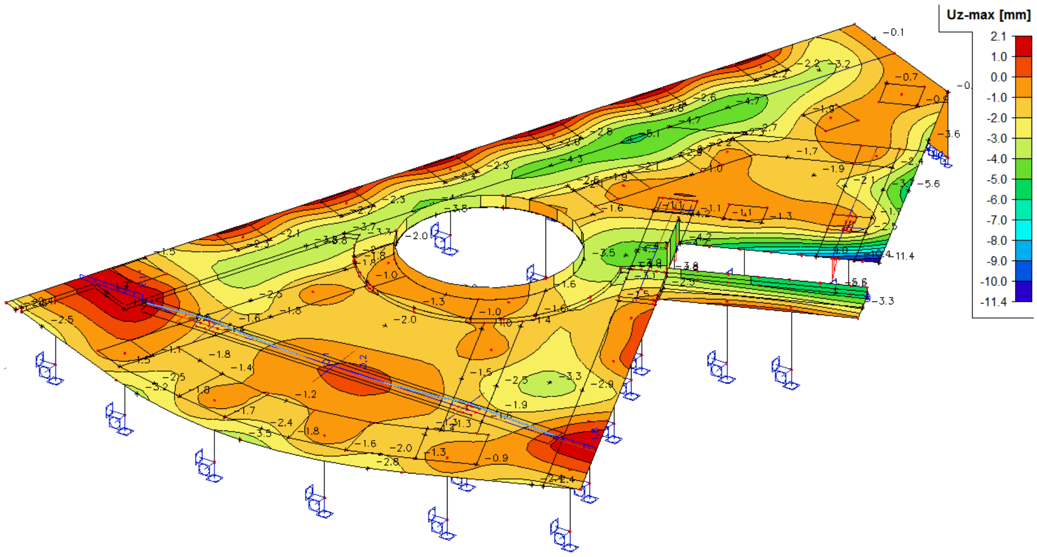



Fig. 8.9 Deflection with maximal lengths of overhangs ($L_1$ = 4.8 m,; $L_2$ = 6.8 m; maximum)

## 8.2 Objective function

Initially the optimization of structure should enable minimal deflection from quasi-permanent combination of the load. Nowadays design of cheap structure is another very important aspect. Generally two views on optimization process exist based on the objective function. This function can be defined for

- minimal deflections and for
- minimal costs.

Generally, this example is typical case of multi-objective optimization task which result to sets of Pareto-optimal solutions. However, a tool for multi-objective optimization has not been developed yet in EOT, the Pareto-optimal frontier can be evaluated after the optimization process. The results are commented in Chapter 8.4.

### 8.2.1 Minimal deflection

The first possibility of optimization is objective function defined based on the minimal deflection. Maximal ($U_{z,max}$) and minimal ($U_{z,min}$) deflection are evaluated for quasi-permanent combination. General shape of objective function is expressed as follows

$$f(x) = \min(U_z) , \tag{8.4}$$

where

$$U_z = \left[\text{abs}(U_{z,\max}); \text{abs}(U_{z,\min})\right] . \tag{8.5}$$

The geometrical constraints of all independent variables were applied for this study (see Tab. 8.1). Additionally, the geometrical constraints get limit the tendon end node geometry. The vertical coordinate has to satisfy the following formulas

$$g_1(\mathrm{x}) = \frac{z_1}{z_2} - 1 \leq 0 , \tag{8.6}$$

$$g_2(\mathrm{x}) = \frac{z_1}{H/2} - 1 \geq 0 . \tag{8.7}$$

Moreover, the value $z_1$ is calculated dependently of the value $\Delta z_1$. Similarly, the value $z_2$ depends on the depth of the girder (see Fig. 8.5 for tendon geometry). An economic effect is evaluated as informative factor only.

### 8.2.2 Minimal costs

As already said, the second possibility is evaluation of total material cost of the girder. Therefore, value $C_m$ is considered as an objective function of hidden girder in term of minimization

$$f(x) = \min\,(C_\mathrm{m}) , \tag{8.8}$$

where

$$C_\mathrm{c} = C_\mathrm{unit,c} \cdot V_\mathrm{c}\,, \tag{8.9}$$

$$C_\mathrm{p} = C_\mathrm{unit,p} \cdot M_\mathrm{p}\,, \tag{8.10}$$

$$C_\mathrm{m} = C_\mathrm{c} + C_\mathrm{p}\,. \tag{8.11}$$

Explanation to the values above: $V_\mathrm{c}$ – volume of concrete in m$^3$; $M_\mathrm{p}$ – total weight of prestressing tendons in kg; $C_\mathrm{unit,c}$ – costs of concrete in 200 €/m$^3$; $C_\mathrm{unit,p}$ – costs of prestressing tendon 4 €/kg.

In addition to this objective function, several constraints are defined. Similarly, geometrical constraints related to vertical coordinates of tendon geometry are used. A verification of deflection is transformed to the side of constraints. Generally, it is impossible to set deflection constraints equal to zero. A limit deflection ($U_\mathrm{z,lim}$ = 3 mm) proved as a reasonable value for deflection constraint expressed as follows

$$g_3(\mathrm{x}) = \frac{U_\mathrm{z}}{U_\mathrm{z,lim}} - 1 \leq 0\,. \tag{8.12}$$

## 8.3 Optimization process

Although, two possibilities of objective functions are defined, the same optimization process is used for both. In general, number of strands is discrete independent variable. Therefore, method handling with this type of variable has to be used. This example is typical case where combination of more optimization algorithms brings better solution than using one method only. Several testing proves the most suitable method modified simulated annealing (MSA) with combination of sequential quadratic programming (SQP). Method of MSA finds optimal number of strands as global optimum and estimates local optima of the overhangs, girder geometry and tendon geometry. While SQP method is usable for continuous variables only, discrete optimal number of strands found based on MSA method was set as constant in the further optimization process.

### 8.3.1 Optimum for minimal deflection

The MSA method is based on 33 bits size population. Totally 15 steps of simulated annealing are used in the optimization process. Moreover, the configuration of parameters is accepted for certain level if the 15 successful solutions are found or maximally 150 solutions run. An applying of MSA method with objective function based on the minimal deflection brings the results mentioned in the Tab. 8.2. The optimization task is constrained by the geometrical limits only. These limits are not strict and give relatively wide space for convergence of the method. Finally, optimization process has been finished after 555 iterations. The evaluation of the iteration on the objective function (minimum deflection) is seen in Fig. 8.10. We can see the minimal absolute deflection 2.335 mm is achieved with 44 pieces of strands. Although, the deflection seems to be acceptable, the cost effect of this solution is not such economical in comparison with the second possibility (see Chapter 8.3.2). The total cost of this girder is almost 13 700 € with minimal deflection 2.335 mm. An improvement of optimum from MSA method by the SQP method does not achieve any decreasing of deflection in this case.

Tab. 8.2 Optimum found for minimal deflection task by MSA method

| Solution no. | $L_1$ [m] | $L_2$ [m] | $n_t$ [-] | $H$ [mm] | $\Delta z_1$ [mm] | $U_z$ mm] | $C_m$ [€] |
|---|---|---|---|---|---|---|---|
| Sol. 1 | 4.128 | 4.952 | 44 | 750 | 150 | 2.335 | 13 639 |
| Sol. 2 | 3.918 | 5.456 | 51 | 680 | 250 | 2.469 | 15 214 |
| Sol. 3 | 3.652 | 5.456 | 56 | 730 | 250 | 2.493 | 17 118 |
| Sol. 4 | 4.240 | 4.880 | 52 | 720 | 220 | 2.561 | 15 780 |
| Sol. 5 | 3.876 | 5.336 | 44 | 750 | 200 | 2.567 | 13 702 |

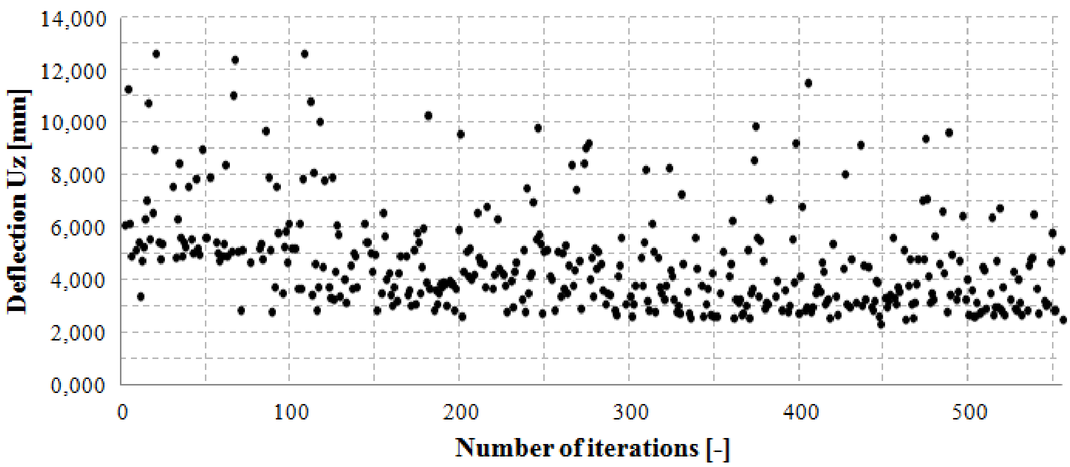



Fig. 8.10 Minimum deflection objective function on the number of iterations

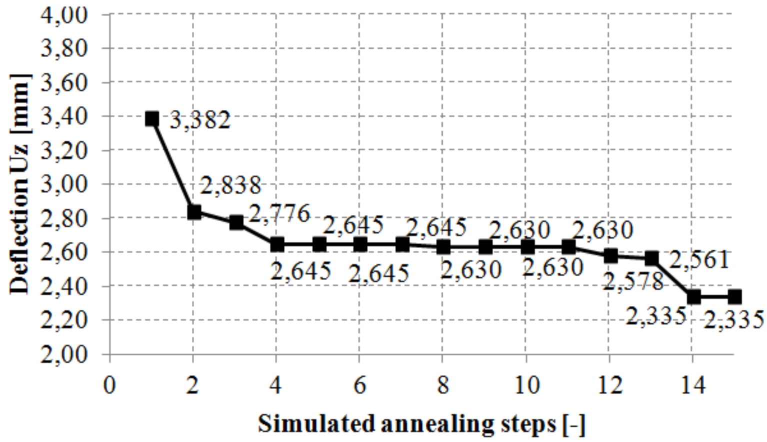



Fig. 8.11 Evaluation of minimal deflection on simulated annealing steps

Finally, the deflection of the slab from the quasi-permanent combination with optimized number of tendon, geometry of tendon and dimensions of the girder is shown in Fig. 8.12.

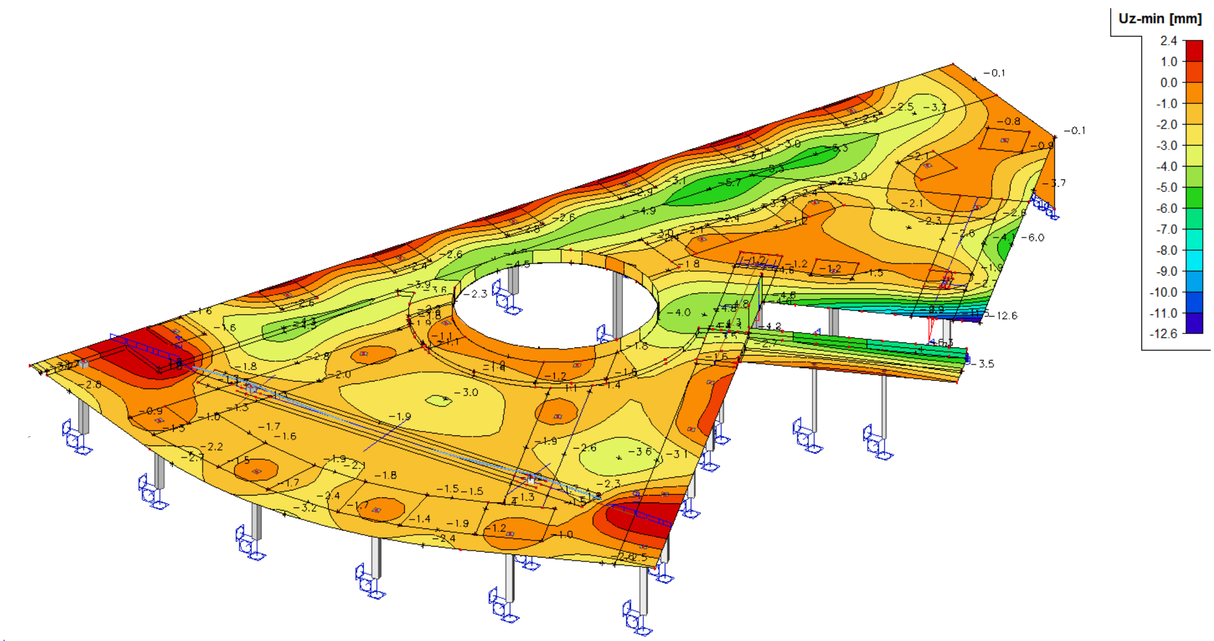



Fig. 8.12 Deflection from quasi-permanent combination for optimal solution

### 8.3.2 Optimum for minimal costs

As already said in Chapter 8.2, two possibilities of optimization are available. The second one is expressed on the minimization of total costs. Again the MSA method based on 32 bits size population is used together 15 steps of simulated annealing. An optimization process found the optimal results of variables summarized in Tab. 8.3. Additionally, two types of constraints are used in this case (see Chapter 8.2.2.). The using of these two levels of constraints causes bigger amount of required iteration for achieving of the convergence criteria. Totally 1950 iterations were needed. The evaluation of the objective function on 15 simulated annealing steps is seen in Fig. 8.13. The minimal calculated objective function is 12 800 € with 2.83 mm negative deflection in the span.

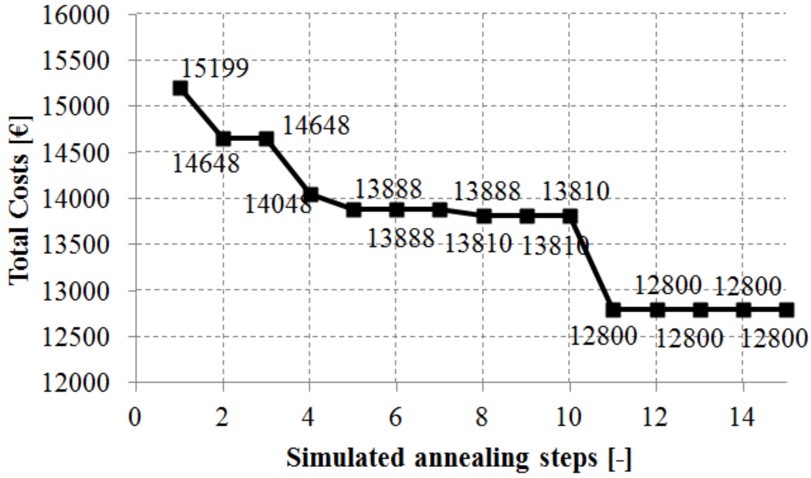



Fig. 8.13 Evaluation of minimal deflection on simulated annealing steps

When the found optimum is improved by more accurate SQP method the minimal costs even decreases. Finally, the minimal costs are 12 050 € (see Fig. 8.14) with 3.0 mm negative deflection. It means saving about another 6% towards to MSA optimum. A displaying of linear deflection of optimal solution from quasi-permanent combination is seen in Fig. 8.15.

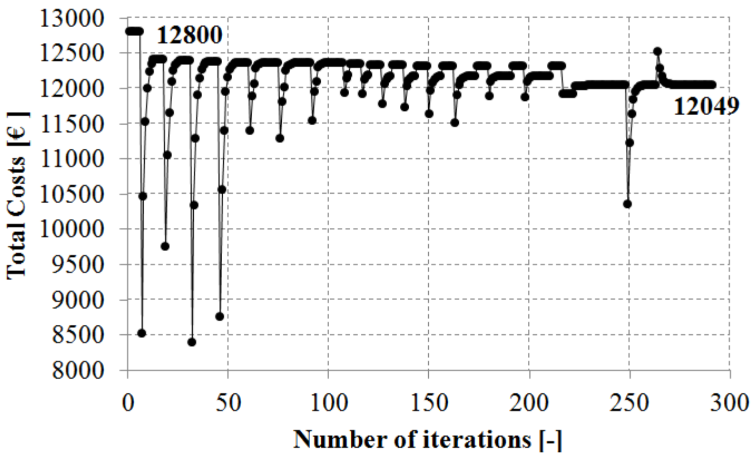



Fig. 8.14 Improvement of the minimal costs using SQP method

Tab. 8.3 Optimum found for minimal costs task by MSA method

| Solution no. | $L_1$ [m] | $L_2$ [m] | $n_t$ [-] | $H$ [mm] | $\Delta z_1$ [mm] | $U_z$ mm] | $C_m$ [€] |
|---|---|---|---|---|---|---|---|
| Sol. 1 | 4.086 | 4.784 | 42 | 720 | 90 | 2.83 | 12 800 |
| Sol. 2 | 4.156 | 5.336 | 45 | 720 | 160 | 2.82 | 13 745 |
| Sol. 3 | 2.140 | 4.712 | 50 | 690 | 70 | 2.97 | 13 810 |
| Sol. 4 | 2.140 | 4.904 | 51 | 670 | 60 | 2.95 | 13 888 |
| Sol. 5 | 4.030 | 2.768 | 51 | 690 | 60 | 2.79 | 13 981 |

Tab. 8.4 An improved optimum using SQP method

| Solution no. | $L_1$ [m] | $L_2$ [m] | $n_t$ [-] | $H$ [mm] | $\Delta z_1$ [mm] | $U_z$ mm] | $C_m$ [€] |
|---|---|---|---|---|---|---|---|
| Sol. 1 MSA | 4.086 | 4.784 | 42 | 720 | 90 | 2.83 | 12 800 |
| Improved by SQP. | 3.980 | 4.848 | 40 | 723 | 50 | 3.00 | 12 049 |

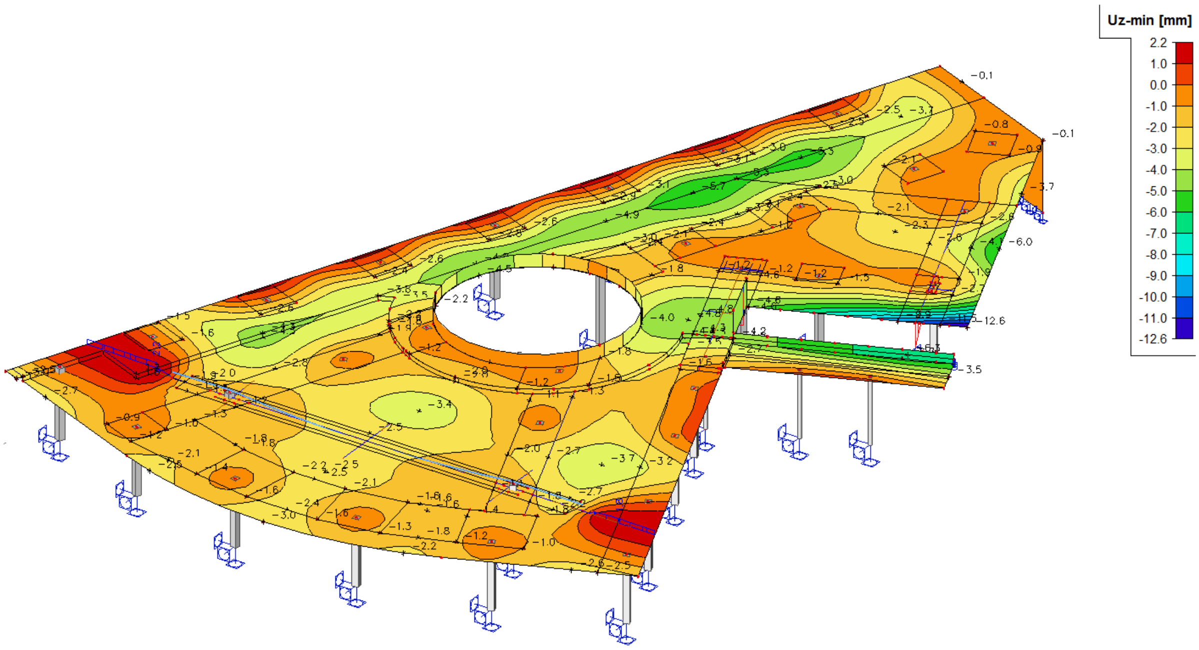



Fig. 8.15 Deflection from quasi-permanent combination for optimal solution

## 8.4 Conclusions

The prestressed hidden girder of concrete floor has been optimized. Initially, the minimizations of linear deflections were required. Nevertheless, the optimization of the structure also focused on the minimization of total costs. Both views found acceptable solutions. The comparison of both methods is presented in Fig. 8.16.

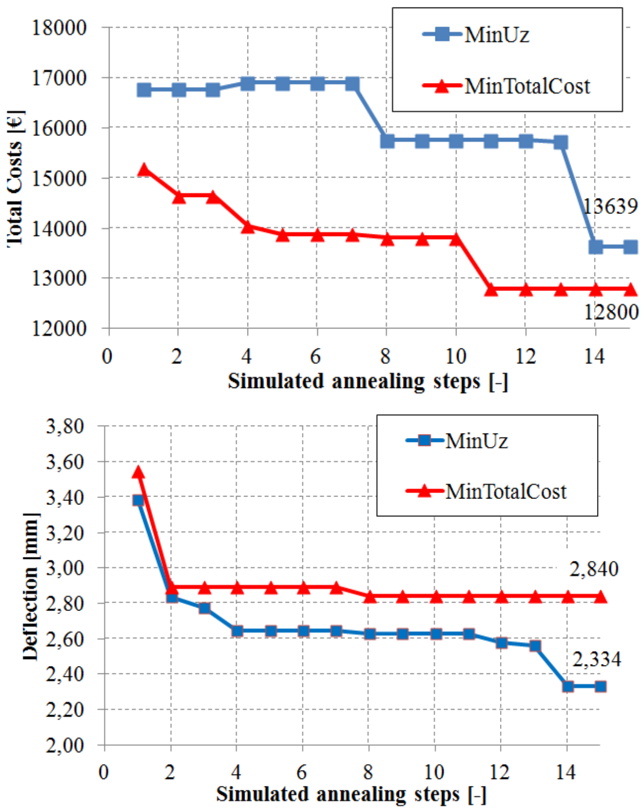



Fig. 8.16 Comparison of both objective function

If the total costs are compared for both types of objective functions, the objective function based on total cost achieves a better optimum. Oppositely, the transforming minimal deflection to constraints is not such strict like minimization of deflection alone.

Generally, both objective functions can be used for reaching of the optimum. The number of iterations needed for optimization is another criterion for the selection of proper objective function. Typically, objective function based on the minimization of deflection is less constrained. Therefore, this approach needs much less iterations than the second possibility (555 iterations for MinUz and 1950 for MinTotalCosts). The optimization based on the minimal Total costs requires almost 4 times more iterations than minimization of deflection. A final decision is composition of mentioned aspects.

When the graphs of minimal deflection dependent on total costs are evaluated, we can clearly see that the example gives set of Pareto-optimal solutions. A list of the acceptable solutions represents what is called the Pareto frontier. The dependency of minimal deflection on total costs for MinUz is shown in Fig. 8.17. Similarly, the Pareto optimal solutions for the objective function MinTotalCost are represented in Fig. 8.18. The curves distributions correspond with the received results from particular optimization (MinUz and MinTotalCost).

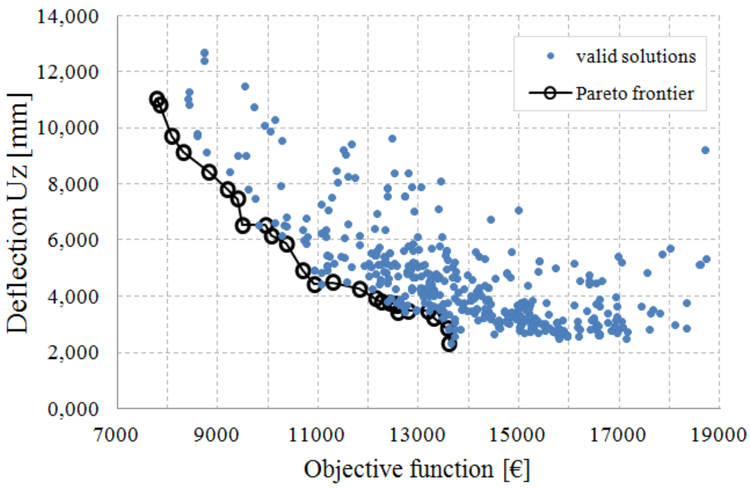



Fig. 8.17 Dependency of minimal deflections on total costs for MinUz – valid solutions within Pareto frontier

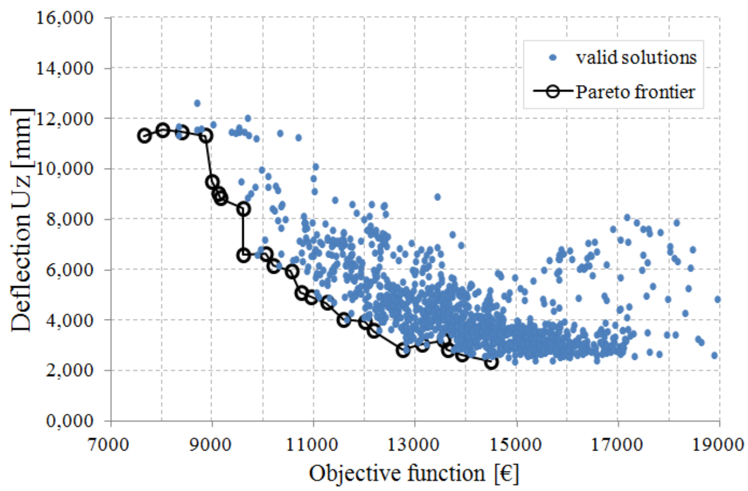



Fig. 8.18 Dependency of minimal deflections on total costs for MinTotalCost – valid solutions within Pareto frontier

Additionally, it can be interesting to know which design variables the most significantly affect the optimal design. Thus, the comparative graphs have been prepared for variant MinUz. As can be seen from the graphs (Fig. 8.19), the most important parameters are number of strands and eccentricity of the tendon at the end of the girder. These variables are almost unchangeable. The remaining parameters do not such influence the solutions on the Pareto optimal frontier.

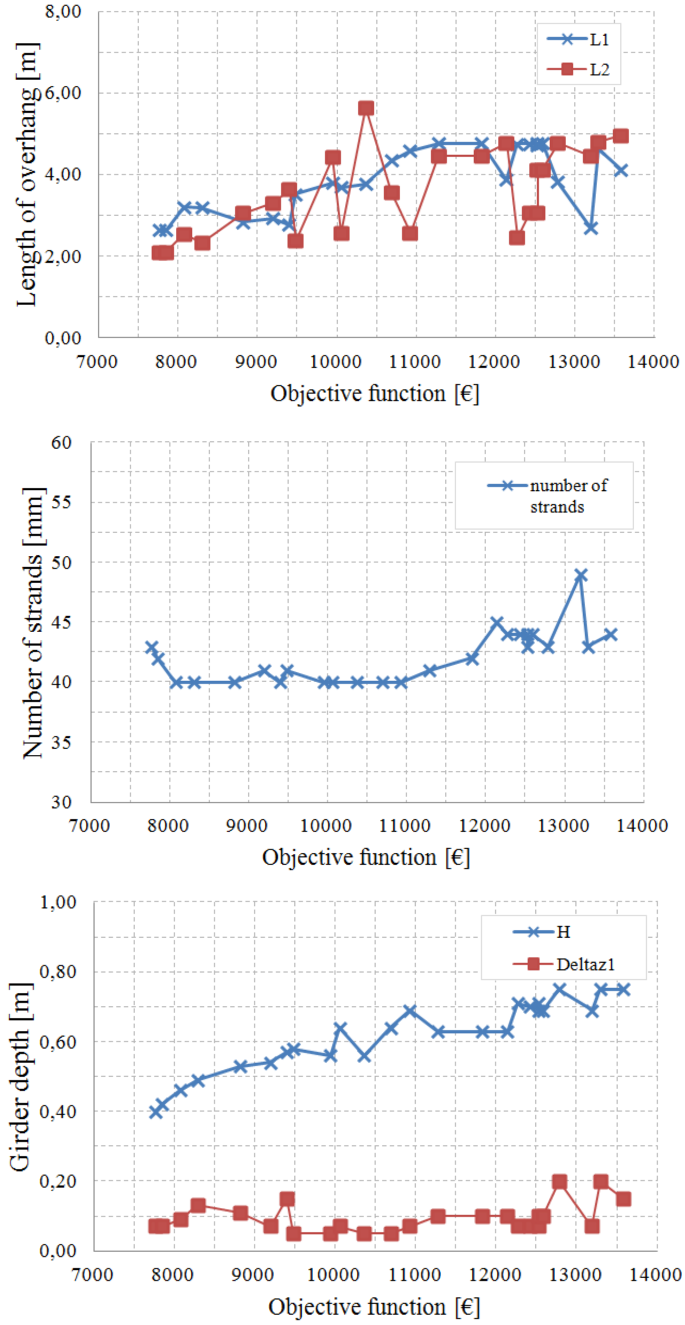



Fig. 8.19 Dependency of design variables on total costs for MinUz on the Pareto frontier

As a final conclusion of this chapter, we incline to minimization of the deflection. In this case, the defining of optimization tasks only based on the defection gives very quality solutions also related to total costs. Especially, the required time for optimization is significantly lesser.

# 9 DESIGN OF CABLE-STAYED BRIDGES USING OPTIMIZATION ALGORITHMS

Cable-stayed bridges belong to large and sophisticated structures used all around the world. They are also very popular from the aesthetic and architectonical point of view. The bridges usually overcome long distances over rivers, deep valleys, etc., where it is not possible to use intermediate supports. Generally, we can distinguish three basic load-bearing elements (i) cables, (ii) deck and (iii) pylons, see Fig. 9.1. None of them can be considered separately, therefore, interaction between the particular elements is required [10]. This system can be varied by changing the pylon shapes and the cable arrangements.

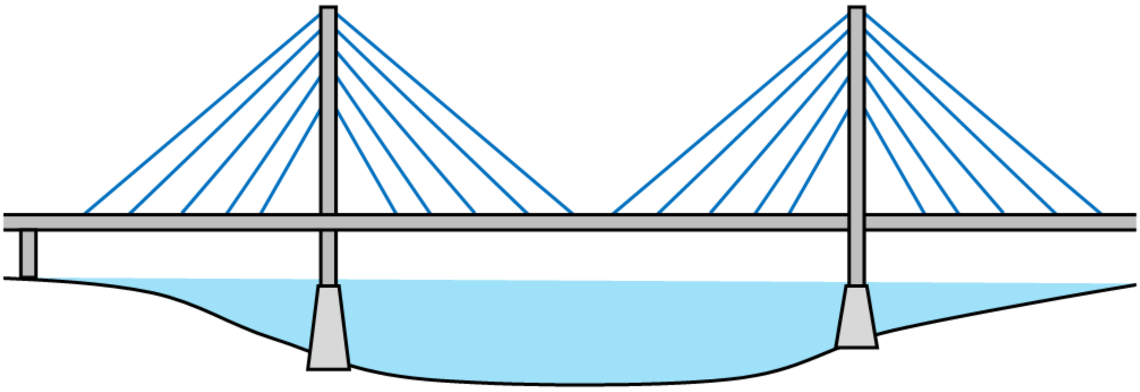


Fig. 9.1 Cable-stayed bridge

The structural behaviour of cable-stayed bridges is based on the tensile forces in the steel cables which produce a compressive stress in the deck and pylons. A model of the deck can be compared with elastic foundation supports in the points of cable-deck connections. The effectiveness of these supports is more evident in case of the axial stiffness of the cables increases. Cable-stayed bridges are usually highly statically indeterminate structures. Hereby, a structural behaviour is influenced by the cable arrangement and cables, deck and pylons stiffness distribution. In addition to the static analysis of dead and live load, the dynamic analysis and that of wind loads, a detailed investigation of the construction sequence is essential.

However, the construction of cable-stayed bridges is exceptional several studies have been already published related to optimization of this kind of structures in the past. Simoẽs and Negrão [109] published article dealing with optimal design of cable-stayed bridge with box girder deck. They developed special computational algorithm enabling optimal design of bridge based on the minimal and maximal stress in cables and deflection limit. Theirs method was based on the pseudo-linear analysis method using Ernst modulus (see Chapter 9.2.2). Sung, Chang and Teo [116] performed the optimal design of the Mau-Lo Hsi cable-stayed bridge in Taiwan based on the engineering approach expressed by the minimal strain energy of the system. The optimization model included restriction related to the displacements of the pylon and limitation of the envelopes of the cable forces. Venkat, Upadhyay and Singh [120] studied possible using of genetic algorithm for design of two pylon cable-stayed bridge. They searched optimal cable forces in stays, ideal dimension of the box deck and tubular pylon. The optimum design was carried out by taking total material cost of bridge as objective function.

## 9.1 Design

There exist many methods for design of cable-stayed bridges. All methods require tensile forces in the cables which reduce the bending moments in the deck and pylons. In that case the deck and pylons are under compression.

A predesign of the initial forces in cables is based on the final structural model. The simplest method for cable forces estimation considers deck like simply supported beam by cables (see Fig. 9.2). It results to preliminary value of a cable area. The different purpose is assuming bridge deck like a continuous beam and the cables provide rigid supports for the deck. Therefore, vertical component of the cable forces equal to support reactions in the continuous beam.

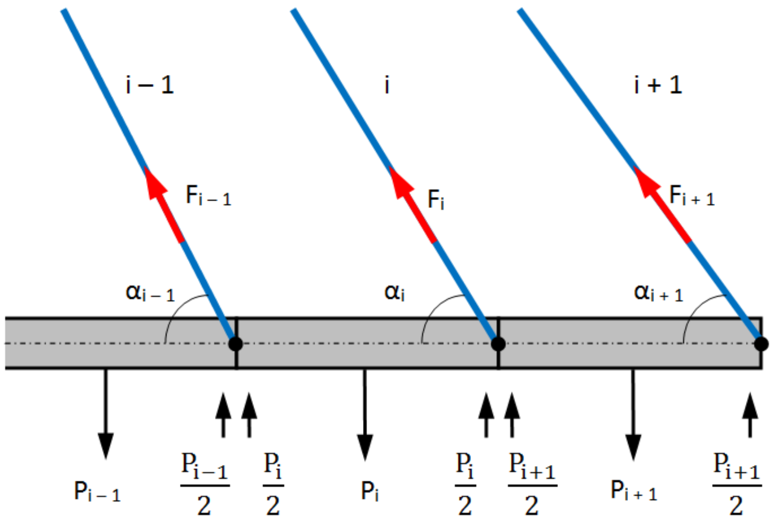



Fig. 9.2 Balancing of deck load by the cables

Generally, the cable-stayed bridges contain a high amount of cables causing the structures very complex and difficult. In this case, it is very complicated to design optimal cable forces to satisfy all necessary restrictions. Therefore, it is almost inevitable to use some more sophisticated method for design of cable-stayed bridges. Several methods have been developed in the past. Let us mention dynamic relaxation method or form-finding method for instance. Unfortunately, the methods are complicated for daily use. We can assume the using of optimization algorithm mentioned in this thesis can bring more effective tool for design of cable forces.

## 9.2 Analysis of cable-stayed bridges

The previous chapter commented available design methods of cable-stayed bridges. Similarly, several calculation methods can be used for analysis. Theirs using depends on the level of the expected results. If we perform initial design it is sufficient to use linear analysis with relatively rough results. Another way of analysis it to simulate construction process by the construction stages analysis. Additionally, this can be extended by the introducing of rheological effect like creep and shrinkage (time dependent analysis). Nevertheless, the cable behaviour is characterised by the sagging effect. Therefore, the most realistic behaviour gives the geometrical nonlinear calculation. The partial replacing of them is the using of linear or staged analysis with modified cable E modulus of elasticity.

### 9.2.1 Linear analysis

Linear analysis is the simplest kind of the calculation how can be predesigned initial values of cables forces. A purpose of this calculation is based on the final scheme of the structure. The cable forces transfer self-weight of the structure within all permanent loads. The results of linear analysis have to satisfy prescribed tolerances of the bending moments and the deflections of the deck and the pylons within purpose of allowable tensile stresses in the cables. Generally, this analysis is the first step of the design.

Generally, the cables are defined using 1D members with very low axial and bending stiffness. The initial stress is not automatically taken into account together with the cable cross-section for linear analysis. This behaviour automatically causes huge deformation of cables it selves and higher deformation of deck and pylon (see Tab. 9.2). Consequently, it is recommended to analyze cable stayed structures by TDA module or use geometrically nonlinear analysis. A module TDA [92] is very sophisticated part of SCIA Engineer enabling effective solving of any kind of the structure within initial stresses caused by prestressing. The applying of TDA and GNL are commented in the next text.

#### 9.2.1.1 Unknown load factor

An analysis of cable-stayed bridge usually starts with finding the optimal cable forces for the final shape of the structure under its selfweight. The MiDAS software [135] introduces special function called “Unknown Load Factor” for calculation of the optimal cable forces. The unit tensile forces are applied for each cable to achieve an optimal state of the structure. When the linear analysis is performed, the programme calculates the effect of the unit tension load on the structure, see Fig. 9.3. Therefore, the unknown load factor is calculated for each cable as a response on the unit tension load. Furthermore, the optimal forces in the cables are determined based on these load factors. Additionally, several restrictions have to be taken into account for optimal forces in the cables. The main constraints are bending moments in the deck, displacements of the pylons and deck, respectively. The bending moment constrains reduce its maximal value by changing moment distribution into shape of a continuous beam. The displacements are restricted to the top node of the pylon and the connection points among cables and deck.

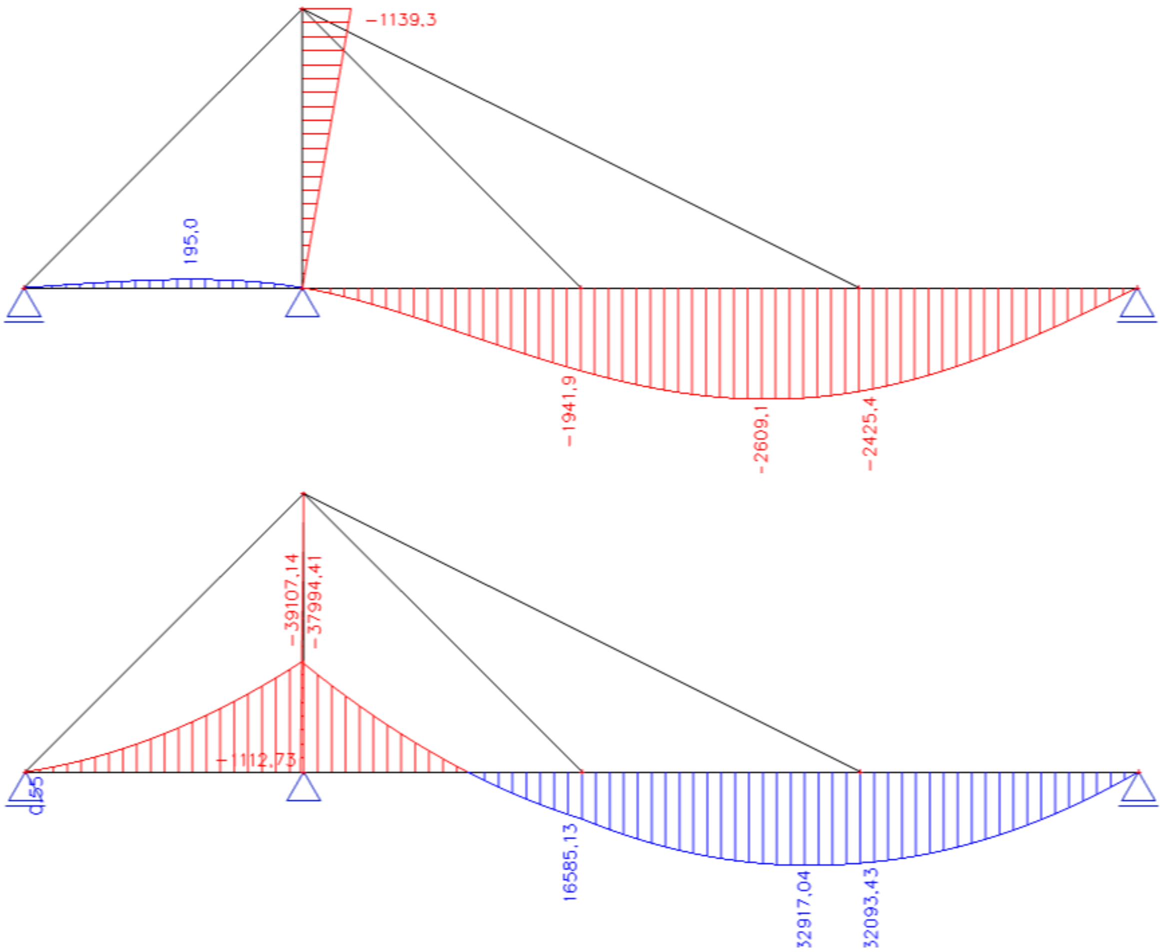



Fig. 9.3 Displacements (top) and bending moments (bottom) on the unit tension load

### 9.2.2 Linear analysis with modified cable E-modulus

As you can read in Chapter 9.2.4, a nonlinear analysis is the best calculation simulating real behaviour of the structure. Sometimes, a required time of nonlinear analysis can be very large. Therefore, the equivalent methods have been developed. One of them is pseudo-linear analysis based on the Ernst modulus replacing nonlinear behaviour of the whole structure by the recalculated cable modulus of elasticity.

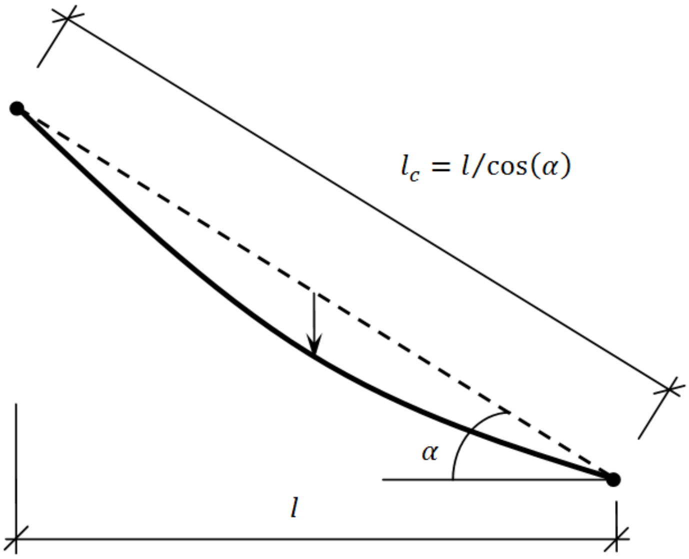



Fig. 9.4 Determination of modified E modulus of a cable

Generally, the axial stiffness of cables depends on two factors, (i) axial deformation of the cable and (ii) the sagging of the cable (see Fig. 9.4). The deformation is calculated based on the elastic behaviour of the cable. The sagging effect causes softening of the cable stiffness. Therefore, the nonlinear calculation is the most suitable analysis method. When the structure is huge, the cables have large sagging and it concludes to relatively low stiffness. Oppositely to small structure where the sagging effect is almost neglected the behaviour of cables became to linear truss element. When we assume elastic behaviour of the cable element during calculation the nonlinear sagging effect of cable complicates the structural analysis. Thus, it is recommended the using of modified tangent E modulus in the linear analysis written as follows

$$\frac{1}{E_{\mathrm{NL}}} = \frac{1}{E} + \frac{\gamma^2 l^2}{12\sigma^3}\,. \tag{9.1}$$

where $E$ is the initial E modulus, $\gamma$ is the weight of cable, $l$ is the horizontal length of the cable and $\sigma$ is the initial stress in cable. As can be seen from the formula above, the modified E modulus consists of initial E modulus representing linear behaviour and sagging effect expressed by the second term. Finally, the formula of modified E modulus can be expressed as follows

$$E_{\mathrm{NL}} = \frac{E}{1+\frac{\gamma^2 l^2}{12\sigma^3}E}\,. \tag{9.2}$$

When we assume elastic behaviour of the cable element during the calculation the nonlinear sagging effect of cable complicates the structural analysis. Therefore, it is recommended the using of tangent E modulus in the linear analysis. For the applying of tangent modulus, the initial cable stresses are required only. Hereby, the calculation comes to simplified and easier to use.

The main disadvantage of this method is that different Ernst modulus has to be calculated for each cable and for particular loadcase based on its stress level. The stresses in the cables vary within certain range. Nevertheless, the minimum tensile stresses are required in the cables to introduce proper stiffness. This limit can be defined as a constraint in the optimization task. Therefore, an average Ernst modulus is efficiently used for each cable, but for all the loadcases. This value considers higher stresses in the cables than real behaviour of the structure.

As you can read in [11], the modified E modulus is important to use for long cables with low initial stress. The cables with horizontal length of 200 m stressed on 400 MPa have values of modified E modulus almost equal ($E = 0.94E_0$). To include Ernst modulus into analysis we have to increase the span length more than 200 m. For this structure, the efficiency of cables rapidly decreases and modified E modulus should be considered. It is recommended to calculate modified E modulus like $E = 0.80E_0$ for bridges with span 400 m and stress in cables equal to 400 MPa. Thus, the cable stayed-bridges with medium long span, where the dead load is significant and the permanent stress in cable is relatively high, the including of modified E modulus can be neglected.

### 9.2.3 Construction stages analysis

What is the characteristic for cable stayed bridge is that process of construction is divided into several stages. A purpose of this calculation is based on the taking the self-weight of one segment by the one or pair of a cables. Span-by-span construction combining the cantilever construction method with suspension by cables belongs to typical processes. Let us assume example from Chapter 9.4 for showing of typical construction stages for this simple example. The process is shown in the following figure.

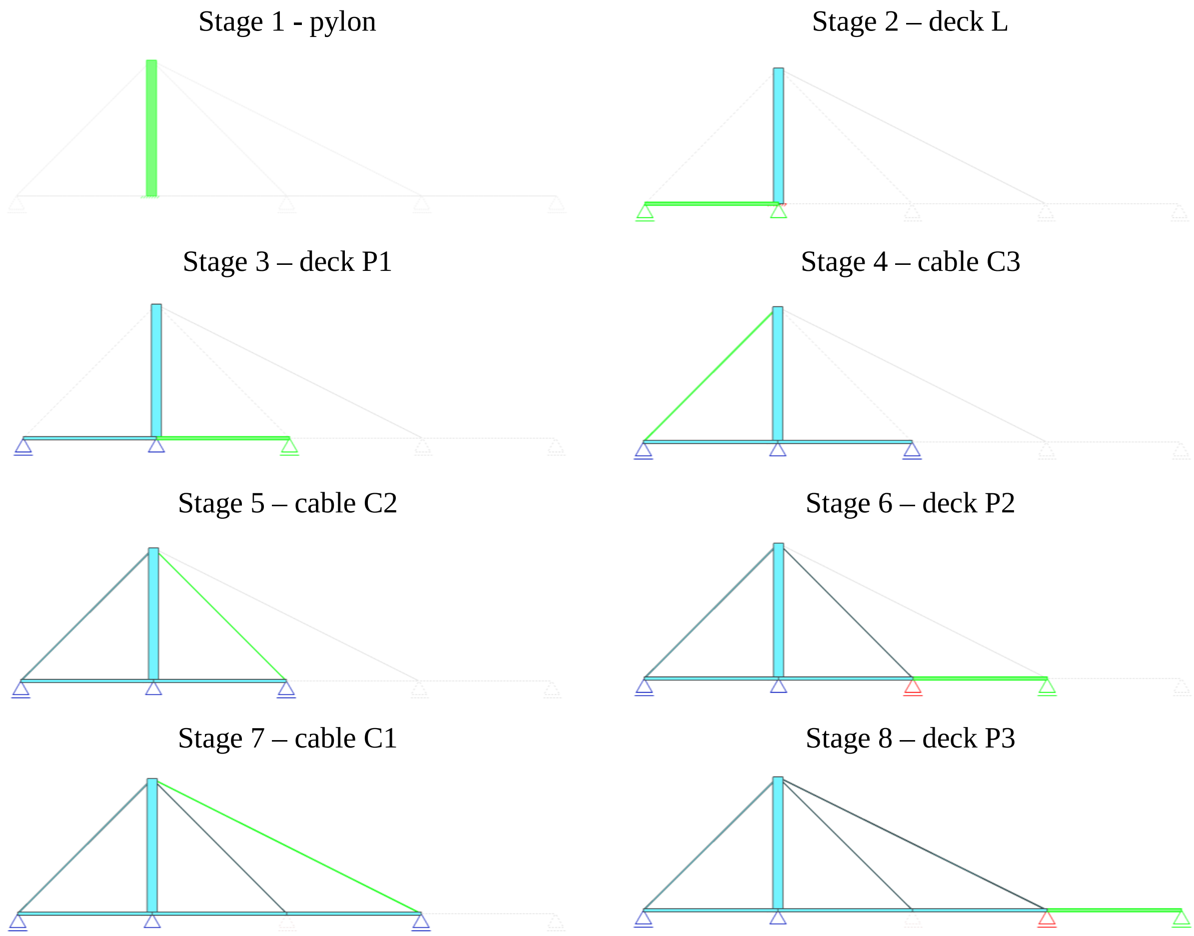



Fig. 9.5 Construction stages of cable stayed bridge

In this case, the optimal design of cable forces does not only focus on final structural scheme of the structure but the verification should be also done for particular construction stages to satisfy all required constraints.

### 9.2.4 Nonlinear analysis

Because of the high deflections of the cables and so called P-Δ effect are typical the cable-stayed bridges are usually solved using geometrical nonlinear analysis. In this case nonlinear cable elements are defined.

When the main span length increases, the nonlinear analysis has to be considered. Additionally, the geometrical nonlinear analysis should be investigated for final and each construction stages which can be complicated for large cable stayed structure with many construction stages.

## 9.3 Optimization

Since the cable stayed bridges are statically indeterminate structures there is no general solution for calculating the initial cable forces directly. Usually it is an iterative process to find an economical solution. These structures may benefit from the use of structural optimization techniques for preliminary design improvement. Generally, it is possible to achieve an optimal state of the structure by the suitable adjustment of the cables. This state typically introduces compressive stresses in the deck and pylons within only small bending effects. A minimization of the total bending energy accumulated along the girder tends to optimal state of the cable-stayed bridges. Hereby, very slender decks from different materials are usually designed. In the previous chapters, we optimize structures based on minimal material costs. Here we are finding particular number of tendons in the cables to satisfy constraints of bending moments in the deck and displacement of the deck and pylon.

### 9.3.1 Objective functions

As has been mentioned above, the optimization of cable-stayed bridge is focused on the finding an optimal forces in the cables. These forces should cause a compression in the deck and pylon within elimination of the bending moment distribution in the deck. Additionally, the displacements can be limited by the maximal values. Therefore, the objective function concerns the minimization of the bending moments distribution in the deck. The maximum value of the bending moments from the maximal positive and minimal negative in absolute values is minimized

$$f(x) = \min\left(\max\left(\mathrm{abs}(M_{\mathrm{y,max}}); \mathrm{abs}(M_{\mathrm{y,min}})\right)\right). \tag{9.3}$$

### 9.3.2 Design variables

An optimized design can be reached by the formulation of the proper design variables. When we optimize a total material cost the dimension of deck cross-section are decreased. Besides, the cable shape and prestressing level are the more important parameters influencing the stress distribution in the whole system. The cable forces are characterized by the number of tendon ($n_{\mathrm{t}}$) in the cable which can be different for particular cable. The maximal cable forces are restricted by the limited value of cable stress ($\sigma_{cable,lim}$). This value is usually considered as follows

$$\sigma_{\mathrm{cable,lim}} = 0.45 \cdot f_{\mathrm{pk}}\,. \tag{9.4}$$

where $f_{\mathrm{pk}}$ is the characteristic tensile strength of the tendon material. Hereby, the design variables are defined.

### 9.3.3 Constraints

When the objective function deals with the bending moment distribution in the deck the other restrictions are transformed to constraints. In general, the constraints can be split to the following groups.

- Displacement constrains – these constraint includes
  - Deformation of the top of the pylon

$$g_1(x) = \frac{|\delta_{\mathrm{p,top}}|}{\delta_{\mathrm{lim}}} - 1 \leq 0 \; . \tag{9.5}$$

  - Deformation of the deck

$$g_2(x) = \delta_{\mathrm{D}} \geq 0 \; . \tag{9.6}$$

- Allowable stress in cables – the minimal and maximal forces are defined using the particular initial cable stresses

$$g_3(x) = \frac{\sigma_{\mathrm{cable}}}{\sigma_{\mathrm{cable,lim}}} - 1 \leq 0 \; . \tag{9.7}$$

- Compression in the deck and pylon – correctly and efficiently designed cable-stayed structure introduces the compression forces in the deck and pylon

$$g_4(x) = N_{\mathrm{P}} \leq 0 \; , \tag{9.8}$$

$$g_5(x) = N_{\mathrm{D}} \leq 0 \; . \tag{9.9}$$

## 9.4 Optimal design of cable-stayed bridge

In the following text, we will explain the optimal design of cable forces of simple example of the cable-stayed bridge using optimization algorithm. The concrete deck from material C35/45 has rectangular cross-section (0.5 × 10 m). The pylon is made from steel (S355) with tubular cross-section 1.0 m width and 50 mm thick tube. A target of this example is to find the optimal cable forces and satisfy all restrictions from constraints. These forces are designed on the final statical scheme as seen in Fig. 9.6. The tendons in the cables consist of the materials Y1770-15,7. A preliminary design considers only the selfweight of the structure.

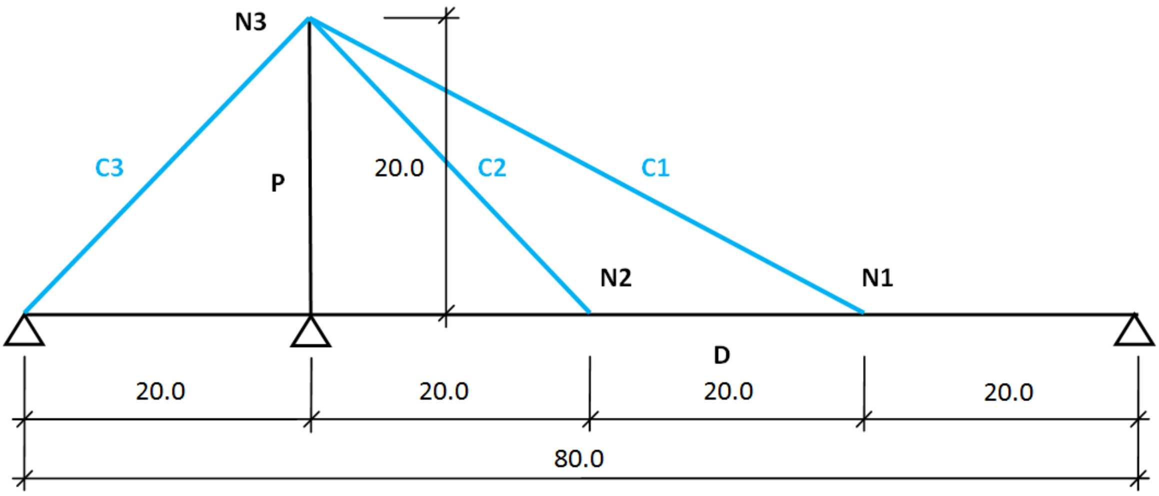



Fig. 9.6 Scheme of cable-stayed bridge example, dimension in metres

Finally, several possibilities of analysis are investigated in details. The comparison concerns the linear analysis with and without Ernst modulus of elasticity, construction stages analysis with and without creep and shrinkage effects and nonlinear calculation.

### 9.4.1 Design variables

As was mentioned above, the optimization focuses on optimal number of tendons in each cable. The initial stress is considered constant based on the characteristic tensile strength of the tendon strand. It concludes to initial stress in the cables equal to 795 MPa. Therefore, the initial stress is not defined as design variable.

$$\sigma_{\text{cable}} = \sigma_{\text{cable,lim}} = 0.45 \cdot f_{\text{pk}} = 0.45 \cdot 1770 = 796.5 \text{ MPa} \approx 795 \text{ MPa} . \quad (9.10)$$

As we can see from the Fig. 9.6, there are three different cable geometries. The design variables concern the optimal amount of the tendons in the particular cable. Hereby, the three design variables are set ($n_{t1}$; $n_{t2}$ and $n_{t3}$). The initial, minimal and maximal values of them are seen from the Tab. 9.1.

Tab. 9.1 Independent variables with limits

| Variable | Initial | Min | Max |
|---|---|---|---|
| $n_{t1}$[-] | 40.0 | 40.0 | 100.0 |
| $n_{t2}$[-] | 20.0 | 20.0 | 70.0 |
| $n_{t3}$[-] | 50.0 | 50.0 | 150.0 |

### 9.4.2 Constraints

When we speak about the constraints, the same ones have been used for the optimization as mentioned in Chapter 9.3.3. The structure has to satisfy the following constraints.

- Displacement – the displacement of the pylon top node and the connection point of the cables in the deck has to comply the established 100 mm limit values
    - Deformation of the top of the pylon

$$g_1(x) = \frac{|\delta_{p,top}|}{\delta_{lim}} - 1 \leq 0\,. \tag{9.11}$$

    - Deformation of the deck

$$g_2(x) = \delta_D \geq 0\,. \tag{9.12}$$

- Compression forces – the compression forces have to be introduced in the deck and pylon

$$g_3(x) = N_P \leq 0\,, \tag{9.13}$$

$$g_4(x) = N_D \leq 0\,. \tag{9.14}$$

### 9.4.3 Optimization algorithm

This type of example belongs to typical case where the evolutionary algorithm should be used. The necessary numbers of tendons in the particular cables are discrete values with defined step of 1 tendon element. The modified simulated annealing is the appropriately selected optimization algorithm. An initial solution for cable consisting of 1 tendon has the results in the following figures. The displacements are in the millimetres and bending moments in kNm. We can see from the Fig. 9.3 that the cables with 1 tendon only are ineffective. The structure behaves like two-span continuous beam with supported pylon.

### 9.4.4 Optimum for particular analysis

As already stated, the modified simulated annealing method is used for the optimization of this structure. The optimization algorithm was applied on examples calculated by the different analysis type. Let us assume the construction stages process shown in Fig. 9.5. Generally, the population consists of 10 members was used for all calculation method. The initial annealing temperature was calculated based on acceptance of 50 % of members. As we can see from the table Tab. 9.2, the optimal results are different for particular analysis method. Nevertheless, the behaviour of the optimal structure tends to the continuous beam scheme where the cable connections to deck represent the internal supports. This effect is clearly visible from the shape of bending moment diagram in Fig. 9.7, which is obtained using nonlinear calculation.

Tab. 9.2 Optimums for particular analysis type

| Variable | LIN | PHA | TDA | GNL | GNL |
|---|---|---|---|---|---|
| | 100 mm | | | | 30 mm |
| $n_{t1}$ [-] | 64 | 54 | 64 | 51 | 44 |
| $n_{t2}$ [-] | 60 | 46 | 58 | 42 | 26 |
| $n_{t3}$ [-] | 148 | 145 | 114 | 106 | 140 |
| $\delta_{N1,N2}$ [mm] | –88.7 | –21.0 | +89.9 | +47.4 | +22.9 |
| $\delta_{D}$ [mm] | –99.3 | –58.9 | –51.4 | +47.4 | –40.5 |
| $\delta_{N3}$ [mm] | –98.1 | –88.7 | –83.7 | +13.0 | +42.9 |
| $M_{y,min}$ [kNm] | –7997.92 | –4119.43 | 7578.50 | –5144.86 | –5075.32 |
| $M_{y,max}$[kNm] | 4887.82 | 4894.03 | –4333.60 | 4136.61 | 4169.77 |
| $f(x)$ [kNm] | 7997.92 | 4894.03 | 7578.50 | 5144.86 | 5075.32 |
| $Length_{P}$ [m] | 8744.32 | 7816.36 | 7726.24 | 6662.16 | 6466.16 |

As has been already discussed in Chapter 9.2.1, the linear analysis is not effective for calculation of cable stayed structure. The statement can be seen in Tab. 9.2, where the deformations after linear or construction stages analysis have high deformation of the deck and pylon, respectively. The initial tensile stress from cables is not automatically taken into account. A stiffness of the system is increased by the truss members doubling the cables. Theirs axial and bending stiffness improve the deflection to acceptable values. The deflections fit the limits for all analysis.

The distribution of bending moments in the deck tends to continuous beam for all analysis type. The least values are for nonlinear analysis above the pylon. The geometrically nonlinear analysis also the most correctly expresses real behaviour of the cable-stayed structure. The usability of the optimization algorithm for nonlinear analysis can be proved the by more strict deformation limits. Thus, the displacements limits are set to value 20 mm, which gives the significantly different results as mentioned in Tab. 9.2. The distribution of the displacements and bending moments based on the nonlinear calculation can be seen in Fig. 9.7.

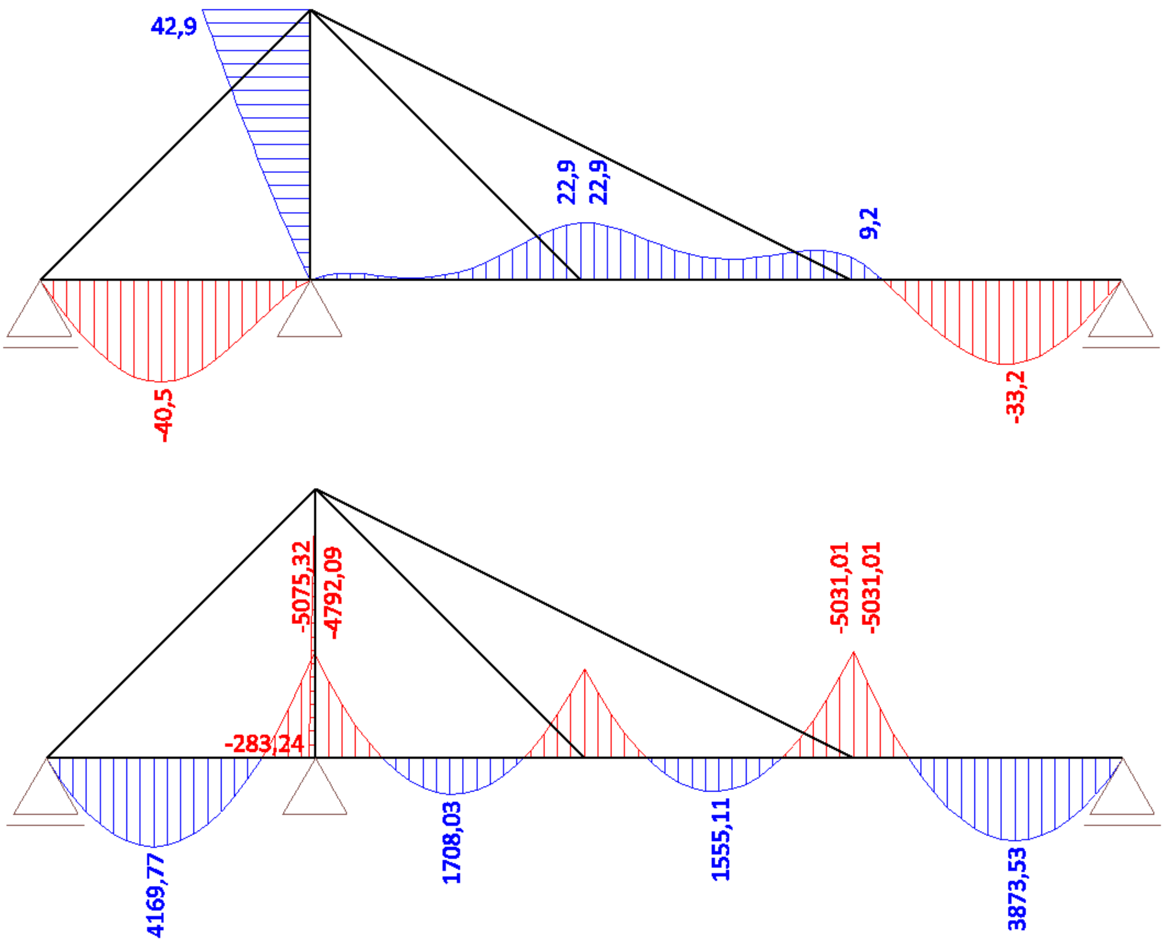



Fig. 9.7 Displacements (top) and bending moments (bottom) of the optimal solution for linear calculation

The comparison of deck displacements is shown in Fig. 9.8 and bending moments in Fig. 9.9. for each particular analysis type. The linear and construction staged calculation express non-realistic distribution of the displacements and relatively high values of bending moments. Especially, a distribution of displacements and bending moments for PHA tends to continuous beam, but there is not effect of the tensioning by cables in the deck. TDA and nonlinear analysis calculate the almost appropriate response of the structure. Unfortunately, the missing geometric nonlinearity in TDA and rheological effect in GNL belongs to main problems in these analysis types. Finally, when we compare the total length of used cable in Tab. 9.2, we can see GNL provide the most economical solution. Nevertheless, GNL can be complicated and time expensive for large statically indeterminate structures with many loadcases and construction stages.

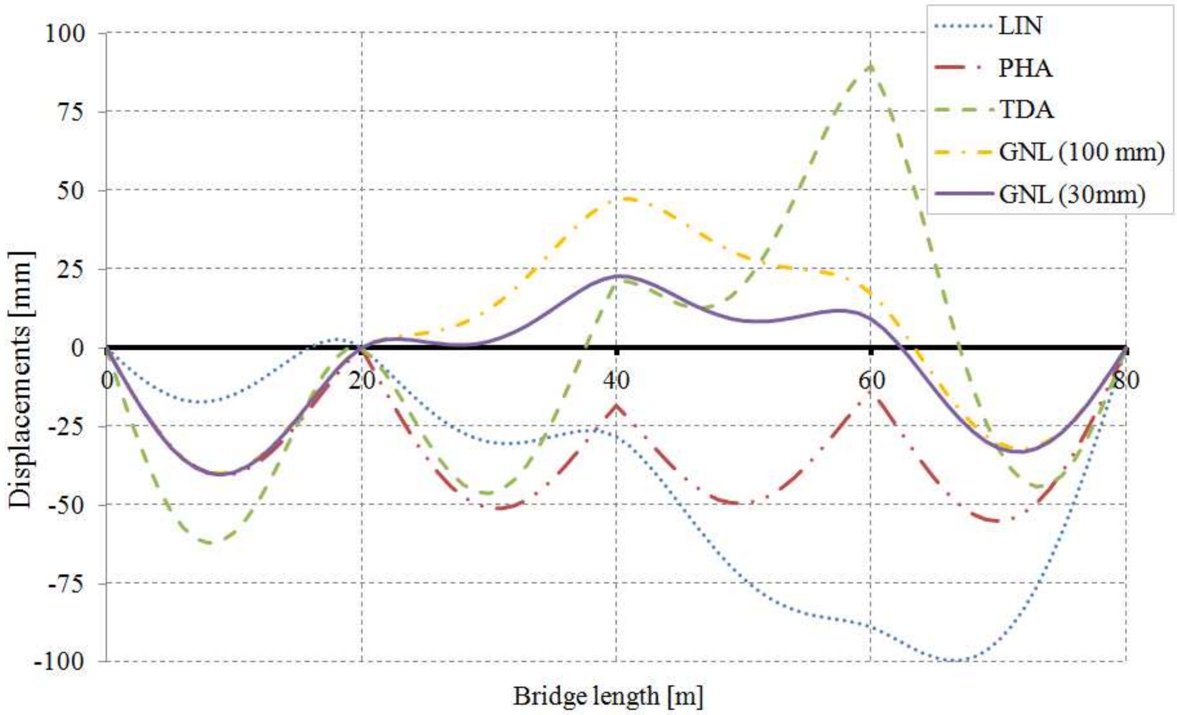



Fig. 9.8 Deck displacements for the optimal solution found by particular analysis type

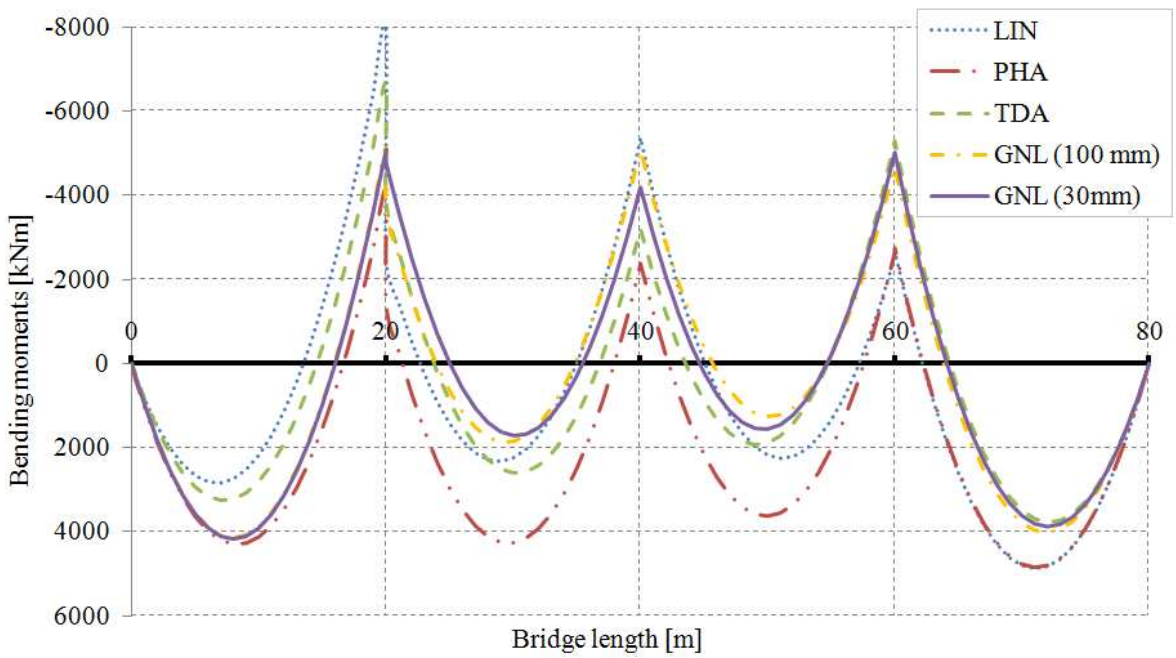



Fig. 9.9 Bending moment of deck for the optimal solution found by particular analysis type

## 9.5 Conclusions

The previous chapters clearly indicate the basic principles of cable-stayed bridges. Several design and calculation methods were described in relation to theirs disadvantages. The optimal design of the cable structure focused on the searching of ideal cable forces in stays. The optimization algorithm was used for design. The method of modified simulated annealing found the optimal number of tendons in cables while satisfying all constraints. The maximal stress in cables is archived by the limit strength in tendon, bending moments distribution is analogous to the distribution along a continuous beam and displacements of the deck and pylon are within the required limits.

As can be seen from the Fig. 9.8, LIN and PHA analysis do not automatically take into account the initial tension stress in cables. The deflections in the connection point among deck and cable deform downwards. On the other hand, TDA and GNL respect the initial stress from the cables. Hereby, the axial stresses of the cables are rapidly increased and the displacements of the deck change upwards. In addition to the stated advantage, the necessary number of cables decrease due to correctly input initial stresses of the cables (see Tab. 9.2). Unfortunately, GNL and TDA are not possible to combine together. The calculation of geometrical nonlinearity in TDA could be very efficient analysis type.

Cable stayed-bridges are often highly statically indeterminate structures. Each cable-stayed structure is unique which leads individualized design of each structure. Therefore, to find ideal cable forces can be difficult also for experienced engineer. We can assume the design of the cable stayed bridge can be improved by the optimization algorithm described in this thesis to have another powerful tool which can help engineers in their daily work. Finally, the chapter showed optimization algorithms do not have to be used only for cost optimal design but also for the ideal design from the standpoint of the statical behaviour of the structure.

# 10 CONCLUSIONS AND RECOMMENDATIONS

The submitted dissertation thesis dealt with analysis of design variables of concrete structures using optimization methods in civil engineering practice. Especially prestressed concrete structures have been studied. The three main goals of the thesis were defined in Chapter 3. The first of them was a studying of optimization algorithms and their implementation to general procedure of optimal design of structure, the second was the verification of proposed method for simple prestressed concrete structure and the last the application of proposed optimization procedure with the most suitable optimization algorithm to several real examples made from prestressed concrete.

1. The studying and testing of the optimization algorithms showed there is no general rule that would always say in advance which method is the best for all optimisation tasks. In the beginning, the four optimization algorithms were analyzed. If we compare the convergence of the method the fast method is SQP. On the other hand, there is the very slowly convergent MSA method. Nevertheless, MSA is very robust method which is able to find the optimum in a wide space of design variables. Obviously, SQP can have a problem with searching of the global optimum. The usability of proper algorithm also depends on the project characteristics. Relatively small projects can be solved by the NM and SQP method. Genetic algorithms have no problem with the optimization of large structures. As was expected, DE and MSA need hundreds of iterations to find the optimum. Consequently, it is clear the universal algorithm does not exist and each method is suitable for different case.

A special tool for analysis of design variables of any kind of structure has been developed [21]. This program enables to repeatedly run finite element calculation of the structure, evaluate the results for particular design variables and set new group of them for next run. The developed optimization algorithms manage the settings of the new group of the design variables and decide when the optimization task is finished. The XML document format is used for the data exchanging between EOT and Scia Engineer [136]. Furthermore, an important aspect is the proper selection of the optimization method. Correct preparatory work is another significant part of optimization. The parameterization of the structure, dependencies among the design variables and theirs limit values can turn out to be crucial.

2. A verification of proposed method for simple prestressed concrete structure proved that only some of them can be efficiently used for the optimization of prestressed

concrete structures. Mathematical programming methods are often very fast but they are sufficient in a limited mathematical space. The simplex based method, sequential linear programming or gradient-based methods are effective for optimization of dimensions of the cross-section, statical model or optimization of prestressing layout only. Additionally, they are limited by using only continuous design variables only. A proper technical solution can be obtained using genetic algorithms like differential evolution or a genetic algorithm. These methods require more iterations but gives better results concerning the optimal diameter and the number of strands or tendons. Thus, the MSA method seems to be the most effective algorithm for the optimization of prestressed concrete structures. This method is very robust and suitable for large structures where we work with dozens of design variables.

Prestressed structures consist of concrete and prestressing reinforcement and include many possible parameters for optimization. Dimensions of concrete cross-section are usually continuous design variables or there are fixed in many cases. Diameter of strands, tendons and number of bars respectively are discrete parameters. Combination of two types of variables concludes to ambiguous optimization method. Fully engineer solution can be obtained using genetic algorithms like differential evolution or genetic algorithm. Methods require more iterations but gives better results of optimal diameter and number of strands or tendons.

3. An application of proposed optimization procedure with the most suitable optimization algorithm have been successfully demonstrated for the design of three types of postensioned concrete like (i) post-tensioned bridges, (ii) post-tensioned slabs and (iii) cable-stayed structures. Each was optimized from a different point of view. The optimal design of post-tensioned bridges focused on the minimization of prestressing material and material cost. Besides, the post-tensioned concrete floor was optimized based on the minimal deflections which were required by the designer. A concern of the cable-stayed structures optimization can be found in minimization of the bending moments in the deck.

Optimization of prestressed concrete structures can be seen from different perspectives. The weight of structures is the one of the main criteria for optimization of concrete structures. Nevertheless, we showed the application of three different objective functions on the design of a post-tensioned concrete bridge. We focused on the objective function based on the minimization of the (i) area, (ii) weight of post-tensioned tendons and (iii) total mass of the structure. As a conclusion, the objective function representing only the area of prestressing reinforcement in one section does not give objective design of structure. The effect of the geometry changes along the beam is not included. Thus, the results can be misleading. Almost each optimization task requires at least one constraint in practice. Nevertheless, not all of them must be included in the optimization. To prevent too time consuming calculation including many checks, it can be efficient to optimize structure only for allowable concrete stresses. As can be seen for the optimization of a post-tensioned bridge, the rational saving depends on the defined type of objective function. The optimization process showed decreasing of objective function about 24%, 17% and 18.5%, for three girders, double T-section and deck bridge, respectively.

The thesis clearly indicated that optimization algorithms do not have to be used only for cost optimal design but also for the ideal design from standpoint of statical behaviour of the structure. The last chapters dealing with optimization of the cable-stayed structure showed that optimization can also be used also for finding optimal configurations of highly statical indeterminate structure. Each cable-stayed structure is unique which leads to an individualized design. Therefore, to find ideal cable forces can be difficult also for an experienced engineer. We can assume the design of the cable stayed bridge can be improved by the optimization algorithm. Consequently, this thesis should introduce another powerful tool which can help engineers in their daily work.

## 10.1 Recommendations for further research

The use of automatic algorithms in civil engineering can certainly bring huge savings. The EOT tool developed as a module to be used in civil engineering practice seems to be promising, even if the total calculation time is not negligible in case of the post-tensioned bridge optimization. The use of parallel computing can be a direction for further development of EOT. Additionally, the implementation of hybrid optimization algorithms can be more efficient. These methods achieve high quality optimums. A combination of the robust genetic algorithm with the accurate gradient based method can be a typical example of a hybrid method. The application of optimization methods on more examples from civil engineering practice will be a valuable contribution to the field. We hope these expectations will be proved so that simple, transparent, and in particular cheaper structures could be designed in practice.

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## LIST OF FIGURES

## LIST OF TABLES

# LIST OF SYMBOLS

## Latin letters

| | |
|---|---|
| $A_c$ | area of concrete cross-section |
| $A_p$ | area of prestressing tendon |
| $A_{p,req}$ | required area of prestressing reinforcement |
| $A_{p1}$ | area of one prestressing strand |
| $B$ | width of cross-section |
| $c$ | concrete cover |
| $C_0$ | overall structural cost |
| $C_B$ | benefits |
| $C_c$ | cost for concrete |
| $C_{constr}$ | cost for construction |
| $C_{er}$ | cost for erection |
| $C_f$ | cost for formwork |
| $C_{FC}$ | functional cost |
| $C_{FC}$ | functional cost |
| $C_{FU}$ | failure cost |
| $C_I$ | regular inspection cost |
| $C_{lr}$ | cost for longitudinal reinforcement |
| $C_M$ | maintenance cost |
| $C_m$ | material cost of the structure |
| $C_p$ | cost for prestressing reinforcement |
| CR | crossing ratio |
| $C_R$ | reconstruction cost |
| $C_{SC}$ | structural cost |
| $C_{SF}$ | cost caused by the failure of the structure |
| $C_{sr}$ | cost for shear reinforcement |
| $c_{Tmax,min}$ | ratio of maximal and minimal annealing temperature |
| $C_{trans}$ | cost for transportation |
| $e_p$ | eccentricity of tendon |
| $F$ | mutation constant |
| $f$ | camber of the tendon parabolic arc |
| $f$(u) | objective function of vector for testing |

| | |
|---|---|
| $f$(x); f(x) | objective function |
| $f_{\mathrm{ck}}$ | characteristic cylinder strength of concrete |
| $f_{\mathrm{ct,eff}}$ | effective tensile strength of concrete |
| $f_{\mathrm{ctm(t)}}$ | mean tensile strength of cornet in time t |
| $g$ | permanent load |
| $g$(x); l(x) | constraint function |
| $H$ | depth of cross-section |
| $L$ | length of the span |
| $L_{\mathrm{p}}$ | length of the tendon |
| $L_{\mathrm{tot}}$ | total length of the structure |
| $M_{\mathrm{Ed}}$ | design value of bending moment |
| $M_{\mathrm{p}}$ | weight of the tendons |
| $M_{\mathrm{u}}$ | capacity of cross-section for bending moment |
| $N_{\mathrm{Ed}}$ | design value of normal forces |
| $n_{\mathrm{g}}$ | number of the same tendon in group |
| $N_{\mathrm{iter}}$ | number of annealing levels |
| $NP$ | number of member in population |
| $n_{\mathrm{t}}$ | number strand in tendon |
| $N_{\mathrm{u}}$ | capacity of cross-section for normal force |
| $p$ | overall acceptance probability of member |
| $P$ | prestressing force |
| $P$(g(x)) | penalization of constraint function |
| $q$ | variable load |
| $s$ | calculated safety factor |
| $s_0$ | limit safety factor |
| $t$ | length of tendon arc tangent |
| $T_0$ | initial temperature during annealing |
| $T_{\mathrm{max}}$ | maximal annealing temperature |
| $T_{\mathrm{min}}$ | minimal annealing temperature |
| $T_{\mathrm{mult}}$ | annealing constant |
| $u_{\mathrm{k}}$ | new vector of design variable in differential evolution |
| $U_{\mathrm{z}}$ | calculated linear deflection |
| $U_{\mathrm{z,lim}}$ | allowable linear deflection |
| $V_{\mathrm{c}}$ | volume of concrete |

| | |
|---|---|
| $V_{Ed}$ | design value of shear force |
| $v_k$ | vector of design variable used for testing in differential evolution |
| $V_{Rd,c}$ | capacity of concrete in shear |
| $X$ | vector of design variables |
| $x$, $z$ | horizontal and vertical coordinates of tendon geometry |
| $x_0$ | initial value of design variable |
| $x_1$ | coordinate of tendon arc vertex |
| $x_i$ | values from vector of design variables |

## Greek letters

| | |
|---|---|
| $\Delta f$ | objective function difference between two members |
| $\Delta\sigma_{c,fat}$ | stress range in concrete |
| $\Delta\sigma_{c,fat,lim}$ | allowable fatigue stress range in concrete |
| $\Delta\sigma_{p,fat}$ | stress range in prestressing tendon |
| $\Delta\sigma_{p,fat,lim}$ | allowable fatigue stress range in prestressing tendon |
| $\alpha_1$; $\alpha_2$ | weighted constant for objective function |
| $\varepsilon_{cc}$ | compressive strain of concrete |
| $\varepsilon_{cu2}$ | allowable compressive strain of concrete |
| $\varepsilon_{pu}$ | allowable tensile strain of prestressing reinforcement |
| $\varepsilon_{tt}$ | tensile strain of prestressing reinforcement |
| $\varphi(t,\tau)$ | creep coefficient |
| $\lambda$ | Langrage coefficient |
| $\sigma_{aa}$ | allowable stress in prestressing tendon after anchoring |
| $\sigma_{cc}$ | compressive stress in concrete |
| $\sigma_{cc,ch}$ | compressive strength of concrete on characteristic combination |
| $\sigma_{cc,lim}$ | compressive strength of concrete |
| $\sigma_{ct}$ | tensile stress in concrete |
| $\sigma_{ct,lim}$ | tensile strength of concrete |
| $\sigma_{lim}$ | allowable stress |
| $\sigma_{max}$ | maximal calculated stress |
| $\sigma_p$ | stress in prestressing tendon |
| $\sigma_{pa}$ | allowable stress in prestressing tendon prior anchoring |

| | |
|---|---|
| $\sigma_{aa}$ | allowable stress in prestressing tendon after anchoring |
| $\nabla f$ | partial derivation of objective function |

## Abbreviations

| | |
|---|---|
| DE | differential evolution |
| EOT | Scia Engineer Optimization Toolbox |
| GNL | geometrically nonlinear calculation |
| LIN | linear analysis |
| MSA | modified simulated annealing |
| NM | Nelder-Mead method |
| PHA | construction stages analysis |
| SEN | SCIA Engineer |
| SQP | sequential quadratic programming |
| TDA | time dependent analysis |

# Appendix

## A STRATEGY SETTINGS OF OPTIMIZATION METHODS IN EOT

The program EOT provides structural optimization of any kind of structure. Generally, there are used methods with different optimization algorithm. The described optimization methods have the specific settings for particular strategy. Therefore, the certain values are defined for each algorithm in this chapter.

### A.1 Sequential quadratic programming

The specific settings for sequential quadratic programming strategy are displayed in the following figure.

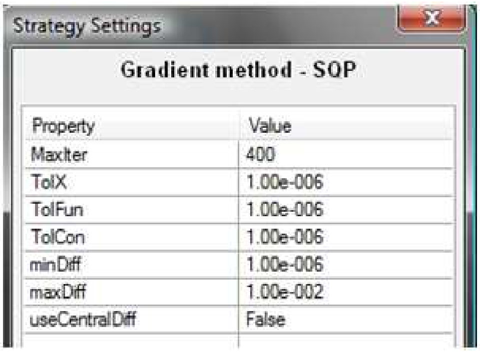
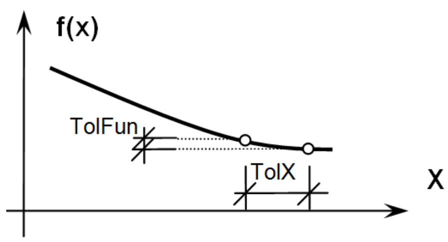



Fig. A.1Strategy settings for SQP method

Tab. A.1 Explanation of strategy settings for SQP method

| Value | Explanation |
|---|---|
| MaxIter | Maximal number of iterations in case of method non-convergence |
| TolX | Minimal distance between the independent variables when the values can be considered as identical |
| TolFun | Minimal difference of objective function for the particular steps of independent variables when the solutions are considered as the same   Fig. A.2 Toleration of design variables and objective function |
| TolCon | Toleration of constraints. For instance, TolCon = $1\times10e^{-6}$ for check value means that solution with check value 1.000001 is accepted |

<table>
<tr>
<td>MinDiff<br>(MaxDiff)</td>
<td>

Minimal (maximal) difference value. The difference ($\Delta x$) is set for each independent variable $x$. Therefore, this is calculated from the range of the particular values. We obtain lower ($d_1$) and upper ($d_2$) boundaries within minimal ($\Delta x_{\min}$) and maximal ($\Delta x_{\max}$) difference respectively. The resultant difference is calculated

$$\Delta x_{\mathrm{E}} = \max(|d_1|; |d_2|) \cdot 10^{-7},$$

where

max $(|d_1|; |d_2|)$ is a maximum from absolute values of lower and upper design variable parameter limit

Afterwards, a difference of design variable corresponding to minimal set difference is

$$\Delta x_{\min} = (d_2 - d_1) \cdot MinDiff,$$

and difference of design variable corresponding to maximal set difference is

$$\Delta x_{\max} = (d_2 - d_1) \cdot MaxDiff.$$

Hereby, calculated value of used difference is the following

$$\Delta x = \max[\Delta x_{\min}; \Delta x_{\mathrm{E}}] \leq \Delta x_{\max}$$

</td>
</tr>
<tr>
<td>useCentralDiff</td>
<td>

Selection option for determining of derivation approximation; it can be True or False. When the option is True then half of difference is offset from the design variable on both sides. In fact, it is approximation by line.

$$\nabla f = \frac{\partial f}{\delta x_i} \approx \frac{f(x_0 - \Delta x) - f(x_0 + \Delta x)}{2\Delta x} = \frac{\Delta f}{2\Delta x}$$

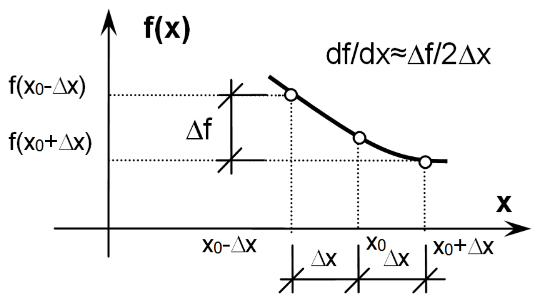



Fig. A.3 Approximation of derivation by parabolic arc (*useCentralDiff = True*)

In case of False, the difference is offset from the design variable on one side only. This characterizes parabolic approximation.

</td>
</tr>
</table>

$$\nabla f = \frac{\partial f}{\delta x_i} \approx \frac{f(x_0) - f(x_0 + \Delta x)}{\Delta x} = \frac{\Delta f}{\Delta x}$$

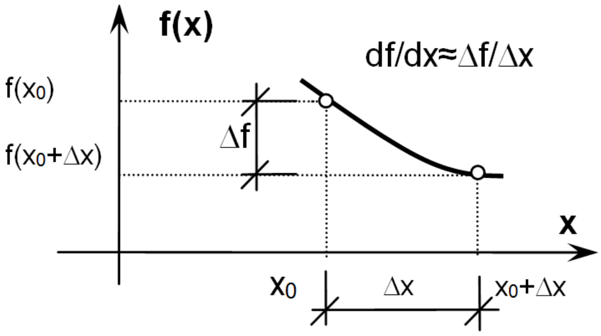



Fig. A.4 Approximation of derivation by line (*useCentralDiff* = *False*)

## A.2 Differential evolution

The Fig. A.1 shows special settings for differential evolution strategy.

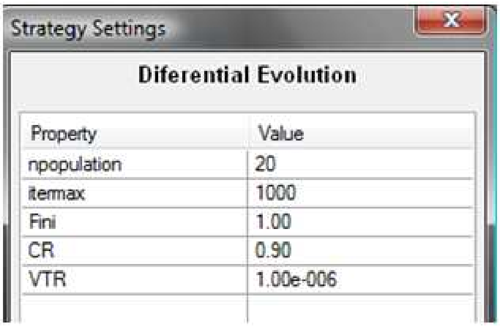



Fig. A.5 Strategy settings for differential evolution

Tab. A.2 Explanation of strategy settings for DE method

| Value | Explanation |
|---|---|
| npopulation | Number of members in population (marked as *NP*). It is required value ($NP > 3$) and recommended ($NP = 10 \times p + 10$) where *p* is number of independent variables. |
| Itermax | Maximal number of iterations in case of method non-convergence for particular population |
| Fini | Mutation constant $F \in \langle 0; 2 \rangle$, this item multiplies a difference of objective function of two randomly selected members. The resultant value is added to third randomly selected member. $$v_k = x_{r3,k} + F(x_{r1,k} - x_{r2,k})$$ When the *Fini* has high value then objective function of $v_k$ is “more different” than two basic selected values |

| | |
|---|---|
| CR | Crossing constant $CR \in \langle 0; 1 \rangle$ |
| VTR | Stop criterion; this setting indicates when the calculated objective function can be accepted as “sufficiently good”. |

## A.3 Modified simulated annealing

The specific settings for sequential quadratic programming strategy are explained in this chapter. As you can see in Fig. A.6, there are more values in comparison with other methods. For practical reason, it is recommended to change the first four values only.

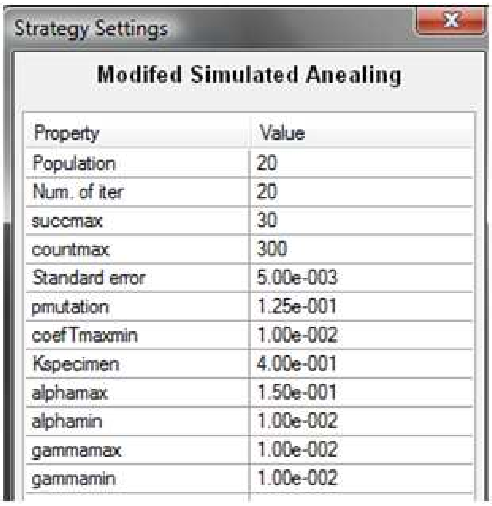
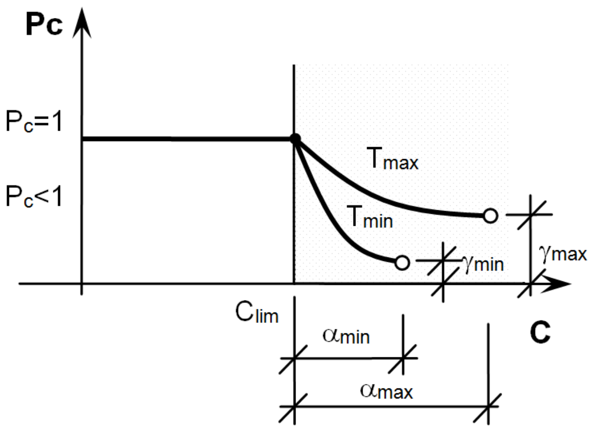



Fig. A.6 Strategy settings for modified simulated annealing

Tab. A.3 Explanation of strategy settings for MSA method

| Value | Explanation |
|---|---|
| Population | Number of members in population; it is recommended this value should be the very close to bit depth of problem |
| Num. of iter | Number of annealing steps |
| Succmax | Number of accepted iterations in certain temperature level |
| Countmax | Maximal number of iteration in certain temperature level which is recommended as follows $countmax = 10 \cdot succmax$ |
| Standard error (SE) | Maximal available error for constrains; when the step of variable is not defined then this value is used for its determination based on the minimal ($d_1$) and maximal ($d_2$) value. $$step = \frac{d_2 - d_1}{SE}$$ |
| pmutation | Mutation probability; this value describes a probability which bit of problem |

| | |
|---|---|
| | gets a mutation |
| coeffTmaxmin ($c_{\mathrm{Tmax,min}}$) | Ratio of maximal and minimal annealing temperature used for the calculation of minimal annealing temperature |
| KSpecimen | Probability ratio of the worst member acceptance towards to the best one. Overall probability of member<br>$$p = p_{\mathrm{f}} \cdot p_{\mathrm{c}}$$<br>where:<br>$p_{\mathrm{f}}$ – acceptance probability of member based on the objective function (see point 4 in Chapter 4.1.2.3<br>$p_{\mathrm{c}}$ – acceptance probability of member based on the constraint<br><br><br>Fig. A.7 Acceptance probability based on the constraint |
| Alphamax | Value of constraint exceeding when the member can be accepted to initial population for the first temperature level |
| Alphamin | Value of constraint exceeding when the member can be accepted to initial population for the last temperature level |
| GammaMax | Acceptance probability of the member even if an independent variables do not satisfy constrain for the first temperature level |
| GammaMin | Acceptance probability of the member even if an independent variables do not satisfy constrain for the last temperature level |

The alpha and gamma values are logarithmically approximated between particular temperature levels.

## A.4 Nelder – Mead

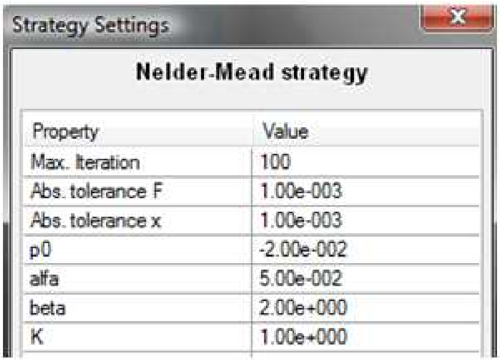



Fig. A.8 Strategy settings for NM method

Tab. A.4 Explanation of strategy settings for NM method

| Value | Explanation |
|---|---|
| Max. Iteration | Maximal number of iterations in case of method non-convergence |
| Abs. Toleration F | Absolute tolerance of objective function |
| Abs. Toleration x | Absolute tolerance of independent variable |
| p0 | Penalization of objective function calculated as follows $$1 + K \cdot \left(\frac{p_c - p_0}{alfa}\right)^{beta}$$ where $p_c$ is penalization of constraint. |
| alfa | Coefficient used for penalization of objective function |
| beta | Coefficient used for penalization of objective function |
| K | Coefficient used for penalization of objective function |